

VOSBURGH

Light Distribution and Efficiency
Tests of a Three Mantle Gas Arc

Mechanical Engineering

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1909

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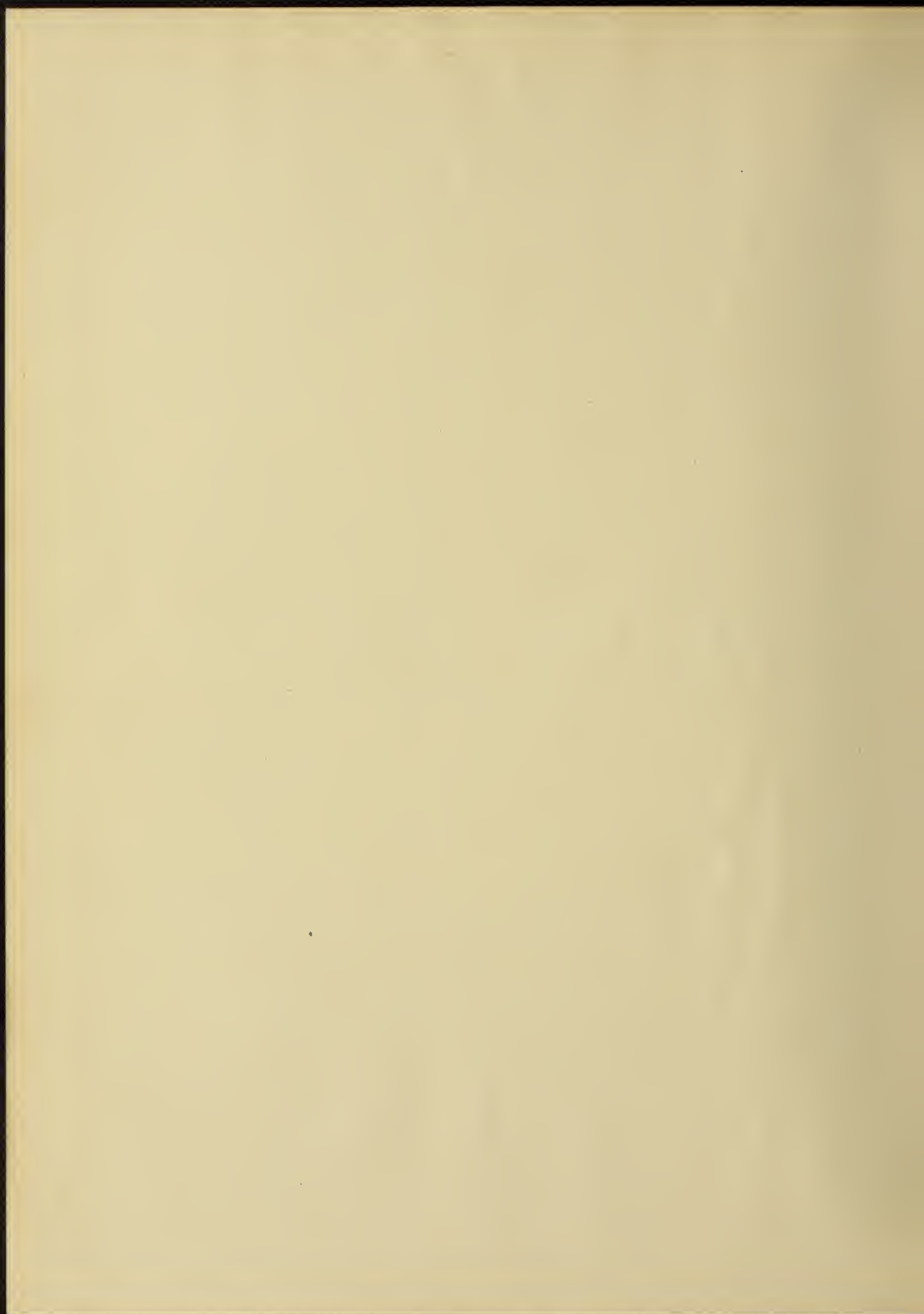
Book

V92

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LIGHT DISTRIBUTION AND EFFICIENCY TESTS
OF A THREE MANTLE GAS ARC

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BY

FRANK JONES VOSBURGH

THESIS FOR THE DEGREE OF BACHELOR OF SCIENCE

IN MECHANICAL ENGINEERING

IN THE

COLLEGE OF ENGINEERING

OF THE

UNIVERSITY OF ILLINOIS

Presented June, 1909

1903
V92

UNIVERSITY OF ILLINOIS

JUNE 1, 1909

THIS IS TO CERTIFY THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

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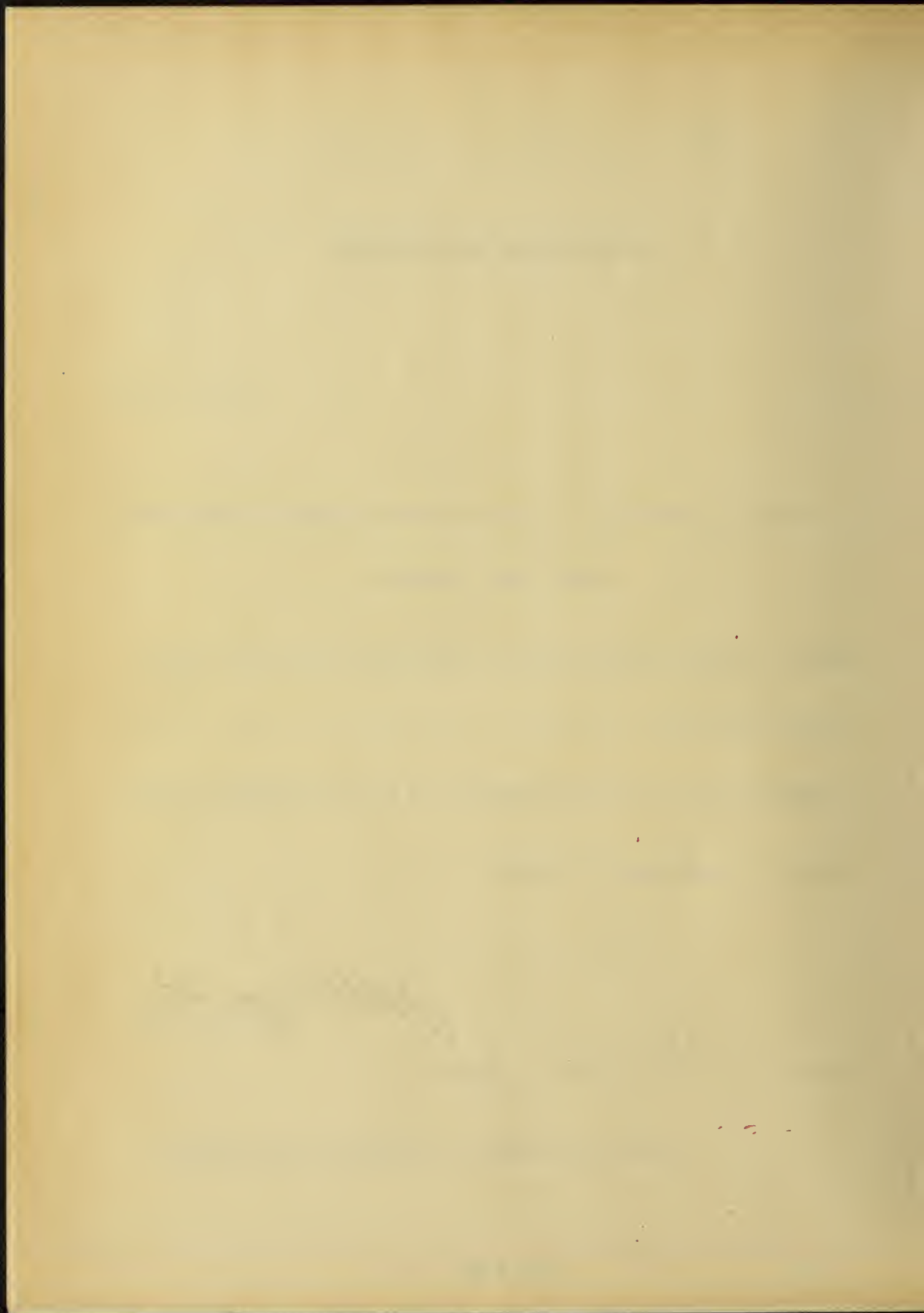
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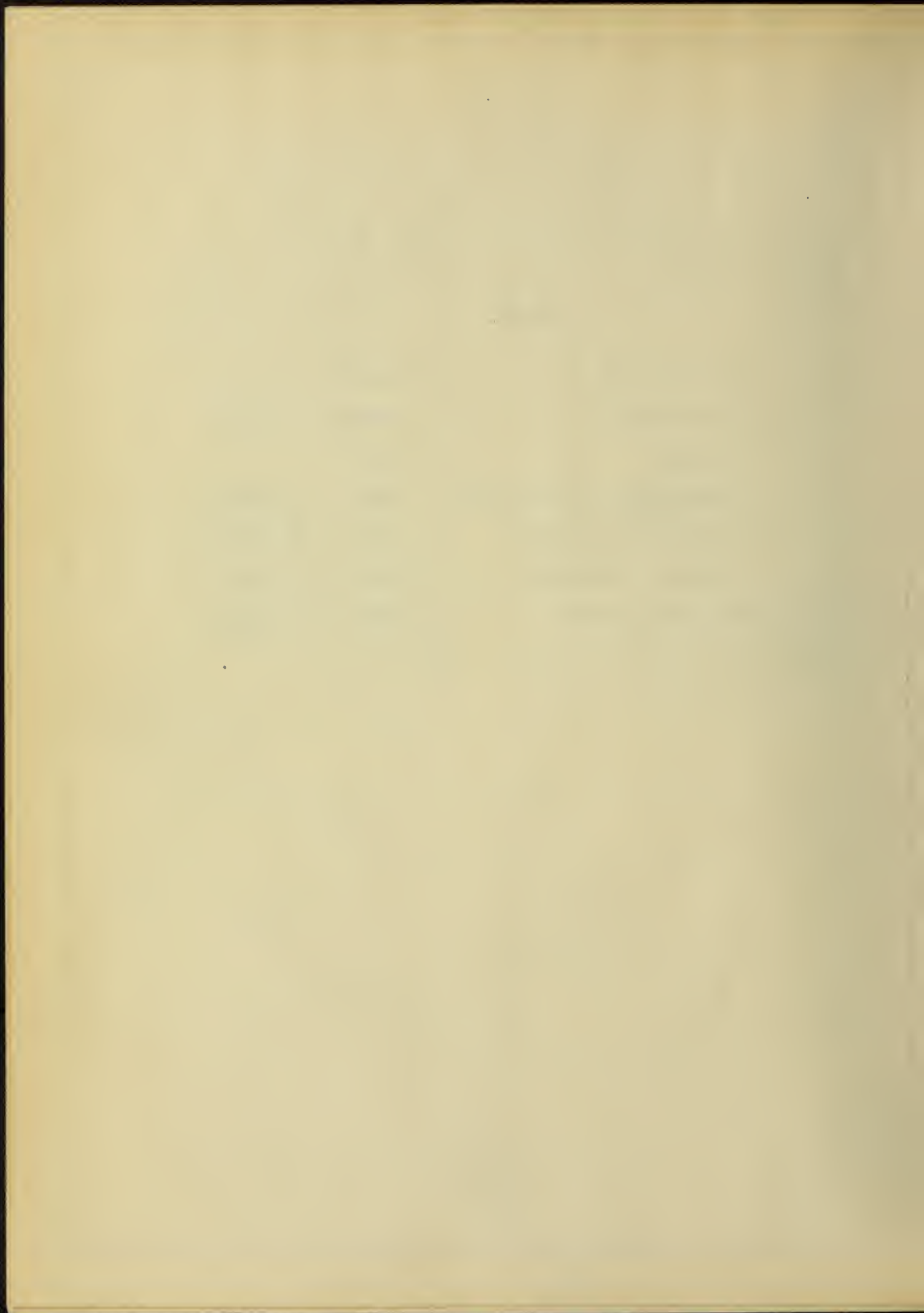
J. M. Bryant
Instructor in Charge

HEAD OF DEPARTMENT OF MECHANICAL ENGINEERING



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Introduction.

The object of the work done for this thesis was to determine the light distribution of a three mantle gas arc, to determine the mean spherical intensity or candle power and from the latter estimate the cost per spherical candle power per hour and compare the same with other forms of illumination.

The work was subdivided into,

- (1) The design and making of the lamp bracket,
- (2) The centering of the light,
- (3) The tests for the coefficient of absorption of the mirror used,
- (4) The tests of the lamp itself,
- (5) The plotting of curves and the determination of the mean spherical intensity.

The thesis would be of greater value, in general, if comparative tests had been run on other lamps of a like description but the author wished only for the results on the one lamp used so further tests were not made.

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Two photographs from different positions, one with and one without the tunnel in place, showing clearly the arrangement of the apparatus and mirror.





A photograph of the lamp when at an angle showing the tunnel of cloth, the gas meter, the mirror and the arrangement of the apparatus and the table



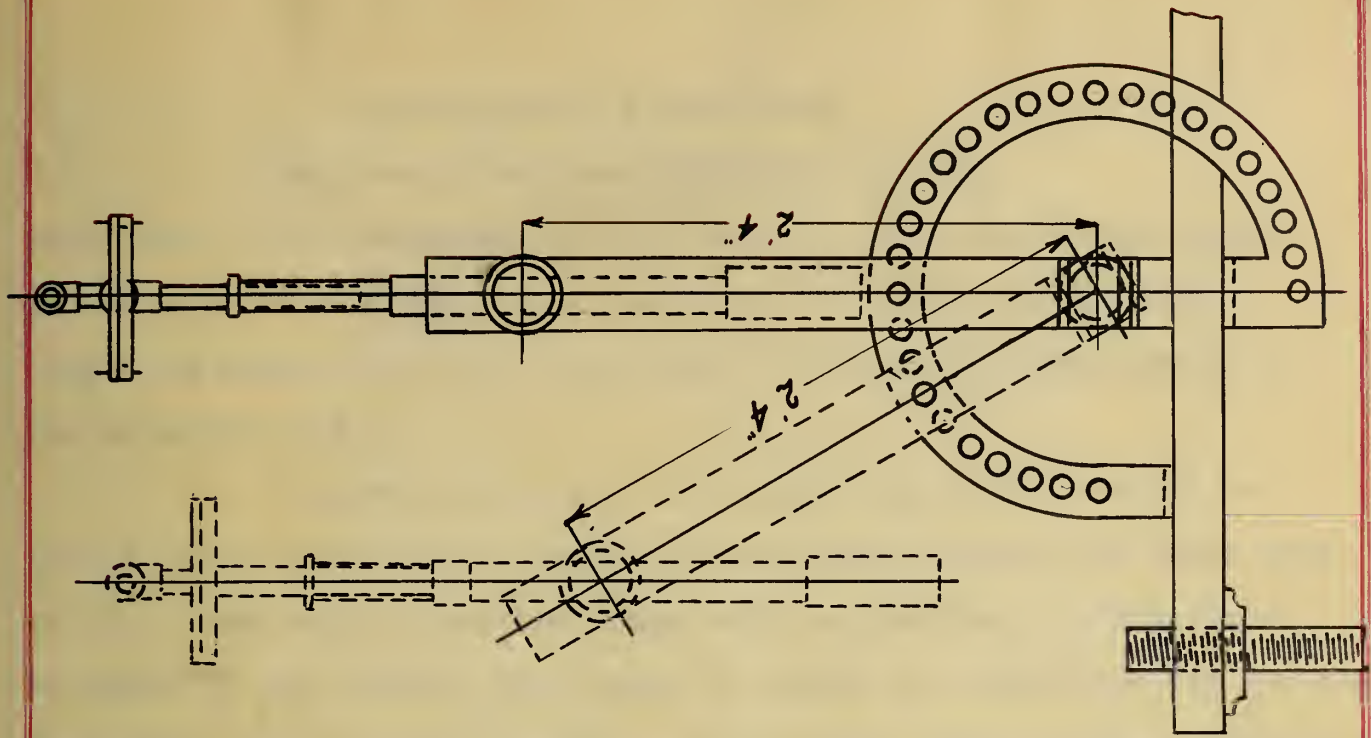
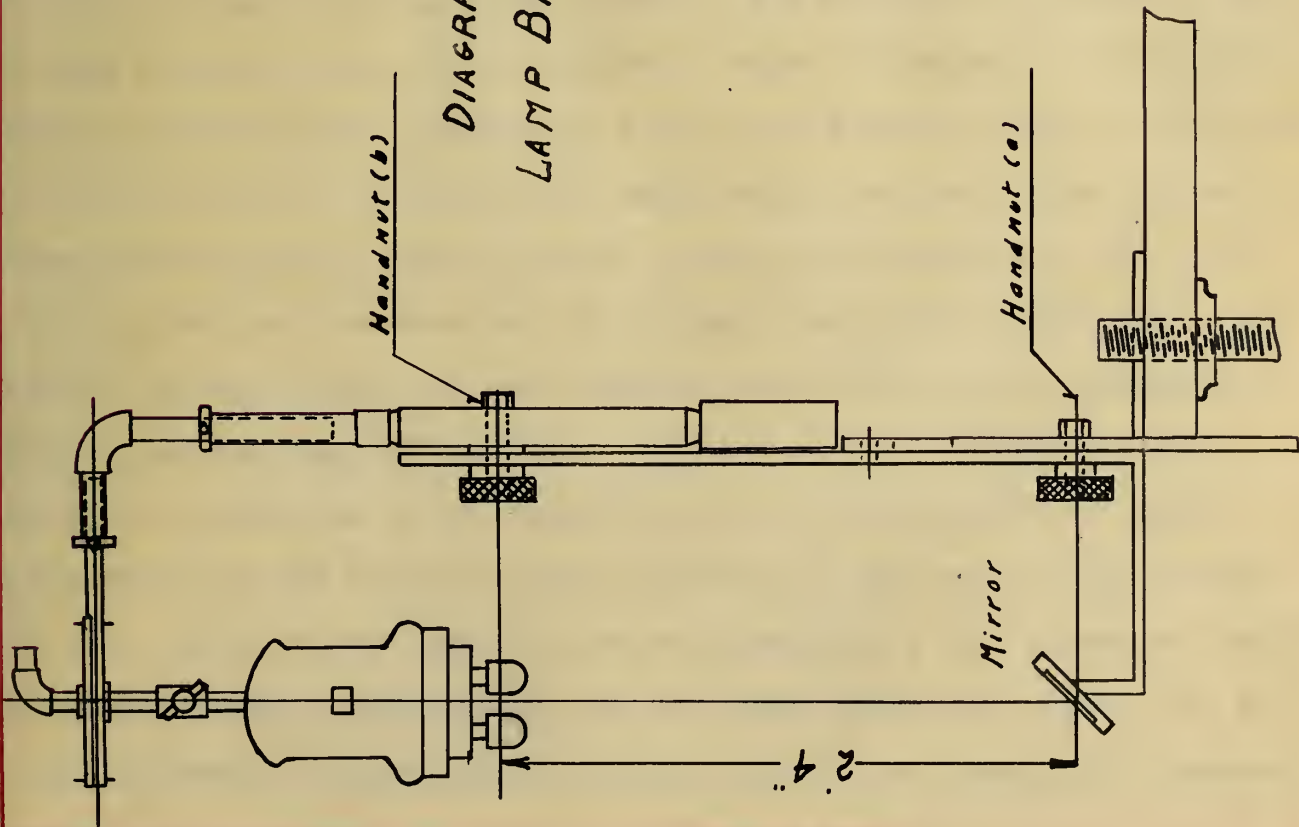
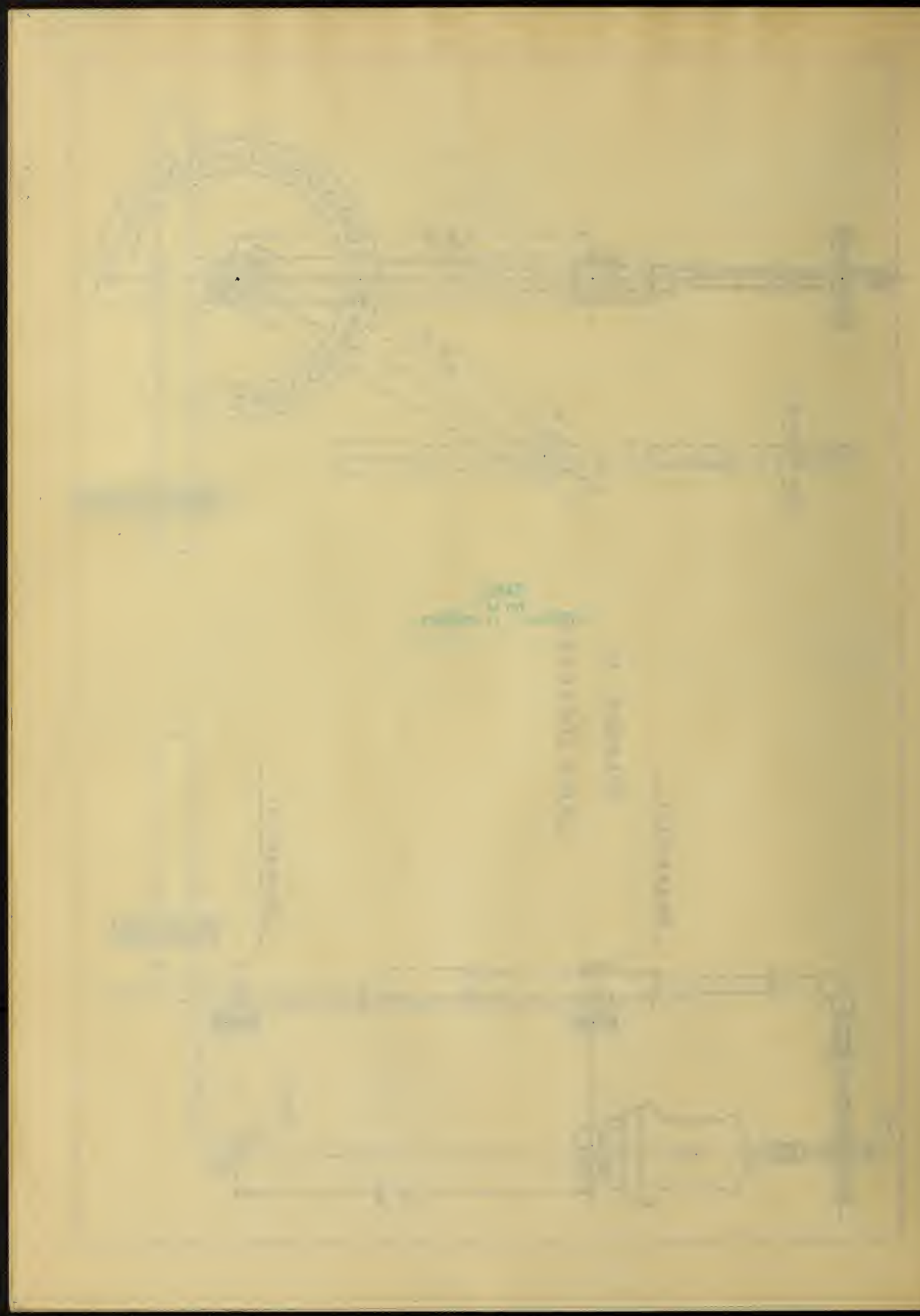


DIAGRAM of
LAMP BRACKET





Description of Apparatus.

The photographs and drawings on pages show fully the arrangement of the lamp and table. These were mounted in one room and the light was reflected from a mirror through a tunnel of black cloth onto the screen of the photometer mounted in the adjoining room.

The table consisted of a heavy plank that rested on iron collars fitting the threads of four gas pipes. On the other ends of the pipes were fitted feet that were screwed fast to the floor. By means of the collars, that could be raised or lowered on the threads the table itself could be moved and the light from the lamp centered as desired.

The lamp bracket consisted of two pieces of flat iron three inches wide and three eighths of an inch thick, one being bent to form a brace, and the other straight and upright for the most part, though a portion was bent, as shown, to form a fixture for the mirror. The two portions were joined by a bolt and knurled hand nut shown at (a). The mirror was mounted on a small plate at forty five degrees from vertical, being held by small screws, and centered on the hand nut (a). The lamp was mounted on another piece, that joined the flat bar, at (b) by a hand nut and revolved about the nut as indicated. This piece holding the lamp was weighted so as to always remain vertical regardless of the position of the bar. In order to permit adjustment of the lamp the piece ended in a gas pipe within which fitted a smaller pipe and the two were held by a set screw and collar. This gave vertical adjustment and the same idea was carried out with the horizontal adjustment. The gas was led to the lamp, after passing



through an accurate meter, by rubber tubing. The tubing joined the short nipple shown and through a cast iron circular plate, that could rotate on an iron ring, and was secured by lock nuts on each side.

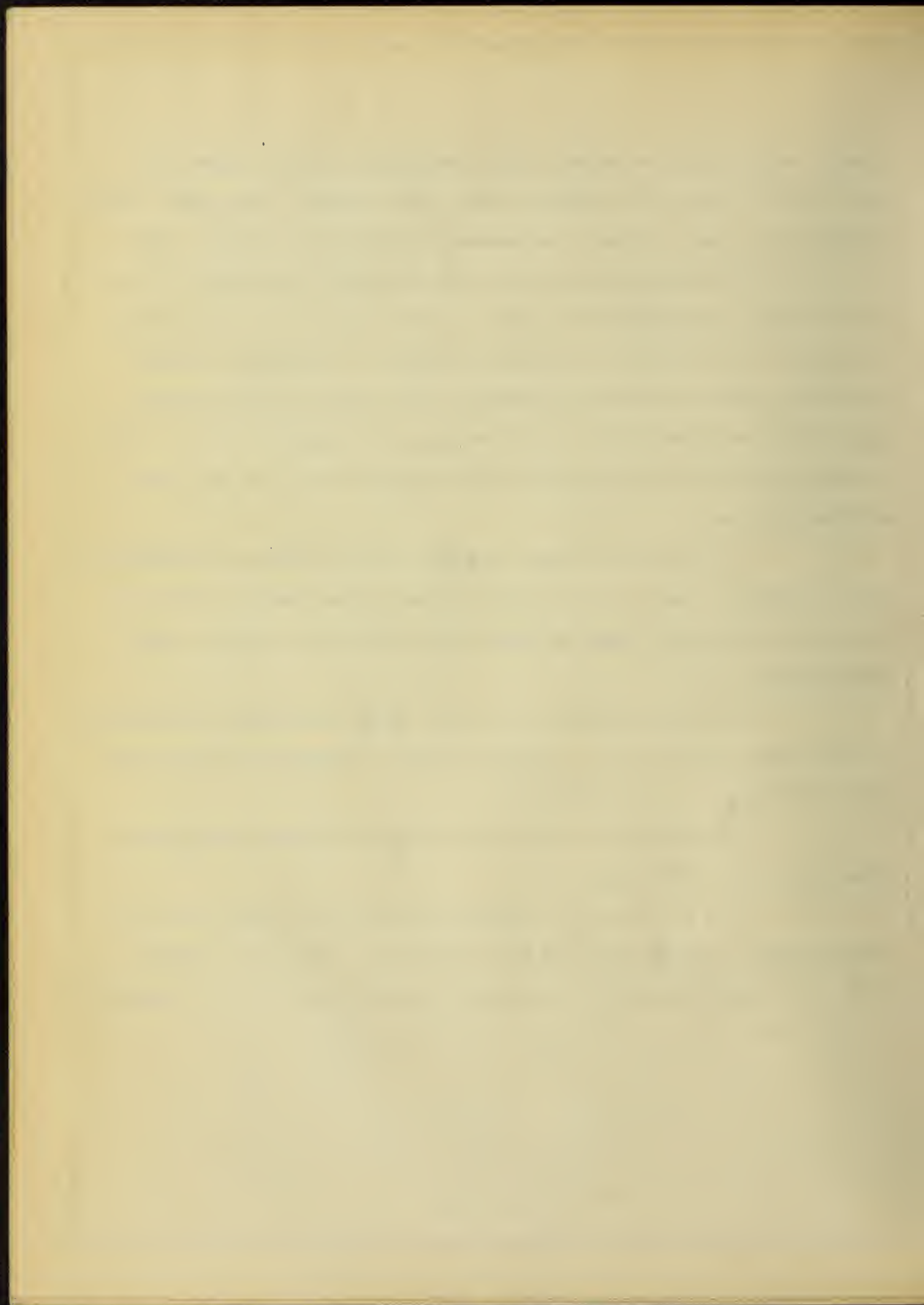
The flat bar could be rotated about the center (a) at one end while the portion carrying the lamp could be rotated about the center (b) as desired thereby being kept constantly vertical and always the same distance separated the lamp and mirror. While light might be taken off of any portion of the lamp's surface since it also rotated in a horizontal plane through the plate and ring mentioned above.

The photometer bar used was but five hundred centimeters long so the lamp was located in an adjoining room making the total effective length from lamp to lamp nine hundred and seventy eight centimeters.

To avoid painting the walls black to prevent reflection of the light the light was directed through a tunnel of black cloth onto the screen of the photometer.

The photometer was of the Lummer Brodhun type made by Queen and Co. of Philadelphia.

The light used on the end opposite the lamp was a 20 candle power tungsten that had been carefully calibrated against a 16 c.p. Standard. The data taken was checked with a 16 c.p. Standard.



Description of the Tests.

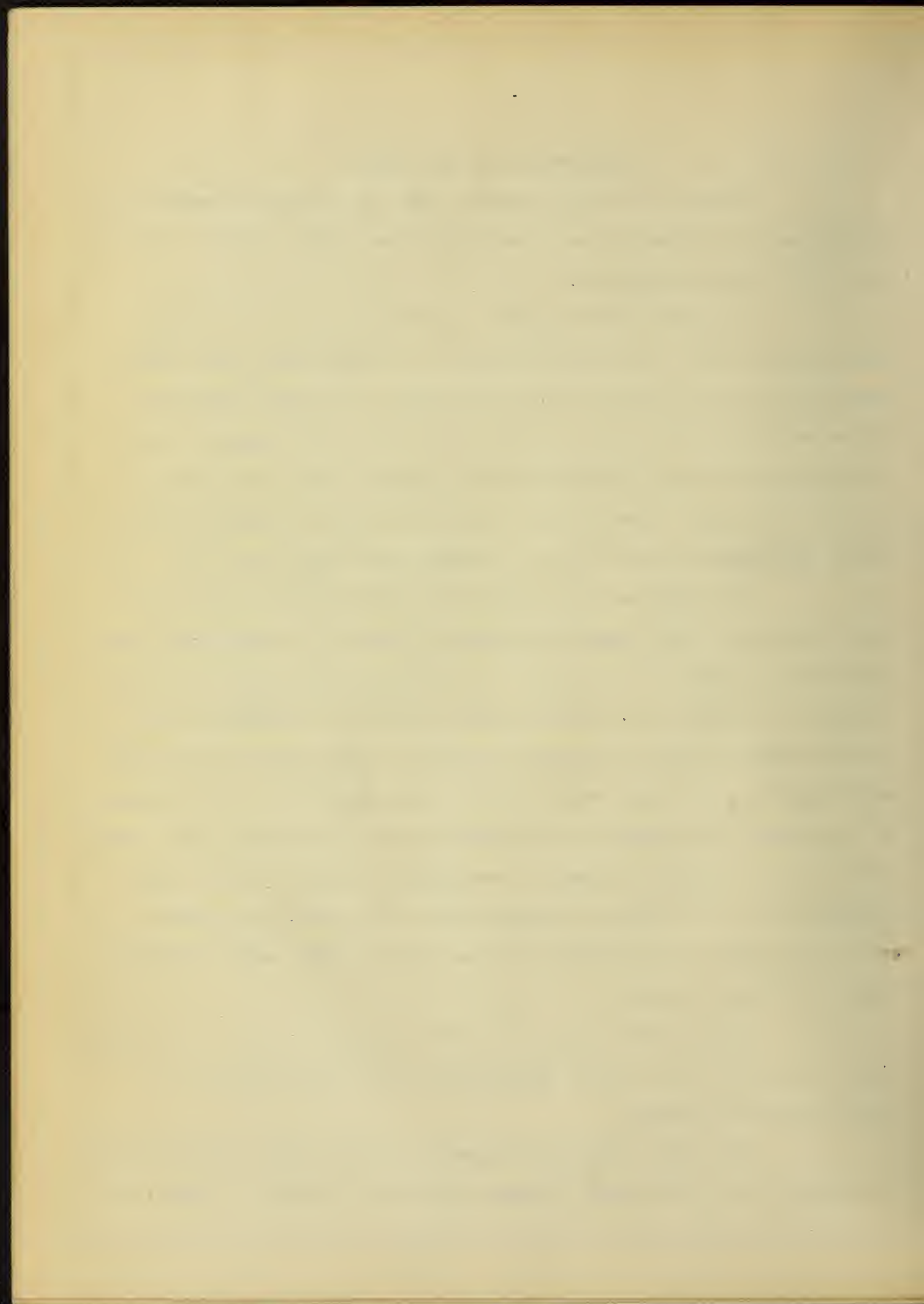
First the light from the lamp was carefully centered on the screen of the photometer by means of the movable collars mentioned and the sliding contacts.

Secondly the tests for the absorption of the mirror were made. In these two electric lights of known candle power were used one at either end, one taking the place of the gas lamp. The lights were balanced on the screen and a series of readings taken and the mean used for the calculations. Knowing what the reading gave for the candle power of the reflected light the ratio of the actual and this gave the factor of absorption which was 1.33 .

The lamp was then tested through every 30 degrees of each horizontal circle and the latter was varied by ten degree intervals from vertical to 130 degrees from vertical. The mean of six bar readings was taken and subtracted from the entire length, 978 cm., and the square of this difference divided by the square of the mean reading since the lights were to each other inversely as the squares of the readings. This gave a factor which was multiplied by the candle power of the lamp and divided by the number of cubic feet of gas consumed and by the factor of absorption thus giving the candle power of the lamp per cubic foot of gas per hour. The sample calculations are shown on page .

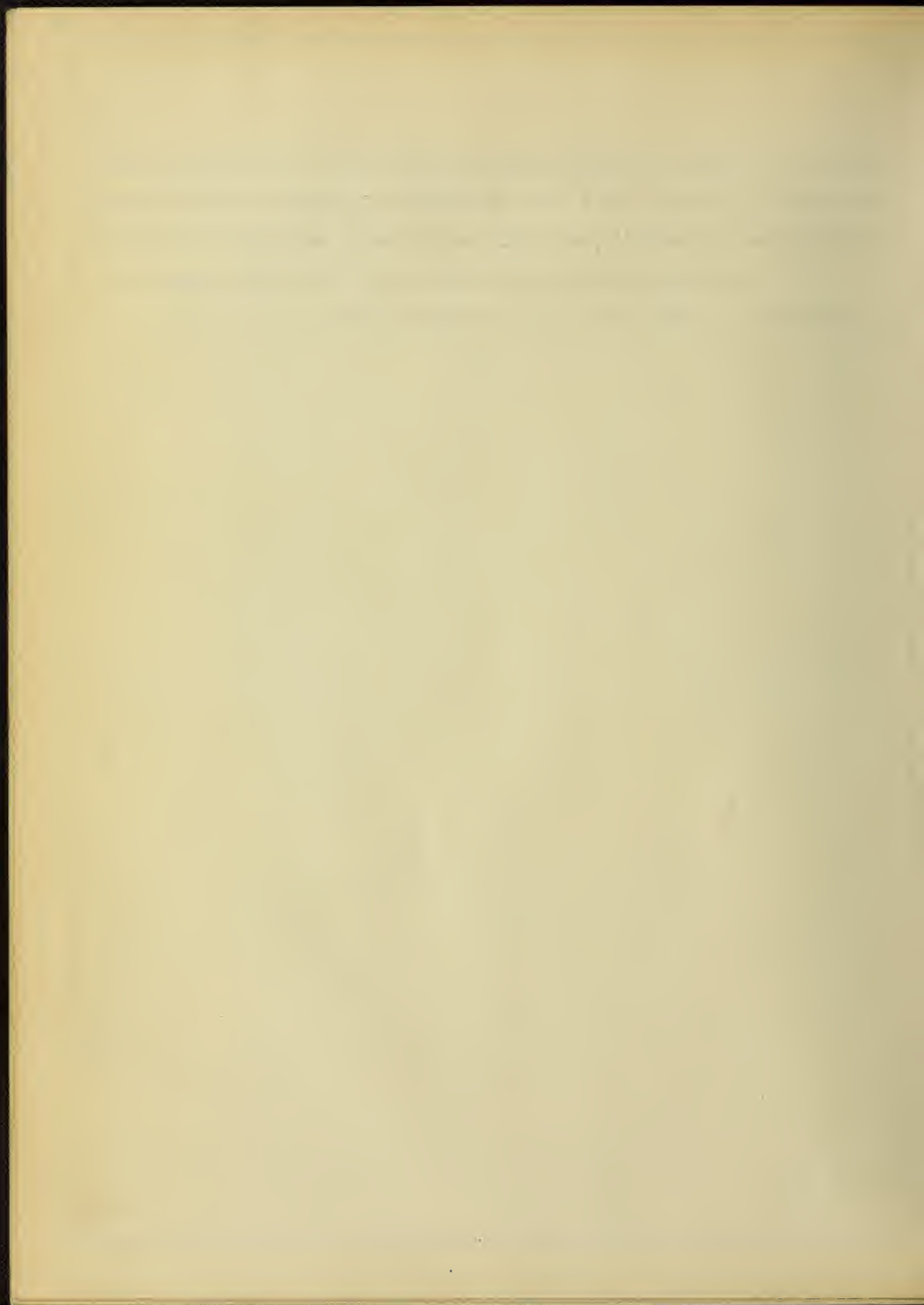
The electric light was run a storage battery the voltage being obtained by the proper manipulation of a potentiometer or "stove pipe" resistance.

The mean spherical intensity of the light was determined by formula and by Rousseaus diagram method. The formula $S = H/2 + M/4$



where H is the mean horizontal candle power and M the maximum candle power, gave as a result 24.4 .The calculation by means of the diagram, which is shown on page 43 ,gave 24.8 as the mean spherical intensity.

A description of the curves on the following pages is not necessary as they are simply the plotted data of the test.



Conclusions.

The tests prove the lamp to be very economical, and for the desired use, the overhead lighting of stores and offices, very nearly ideal since the greater portion of the light is thrown either vertically or nearly so. The result obtained for the vertical illumination, 45 candle power per cubic foot of gas per hour, is very high, exceeding the usual values in some instances by as much as fifty per cent.

The curves show with absolute clearness the exact distribution of the light in every direction, far better than it can be told. The horizontal circles vary, as would be expected, the largest one being that for the vertical position and the smallest when 130 degrees from vertical.

With gas at one dollar a thousand cubic feet the cost per spherical candle power per hour is .004 of a cent. With electric light at six cents per Kw hour the cost per candle power of a carbon light is .02 of a cent and of a tungsten light .007 of a cent or at fifteen cents per Kw hour the former costs .05 of a cent and the latter .0187 of a cent. Thus it may be seen that the gas lighting is much cheaper than the electric lighting in these forms. The cost can not be compared with arc light since the latter is sold on a different scale.

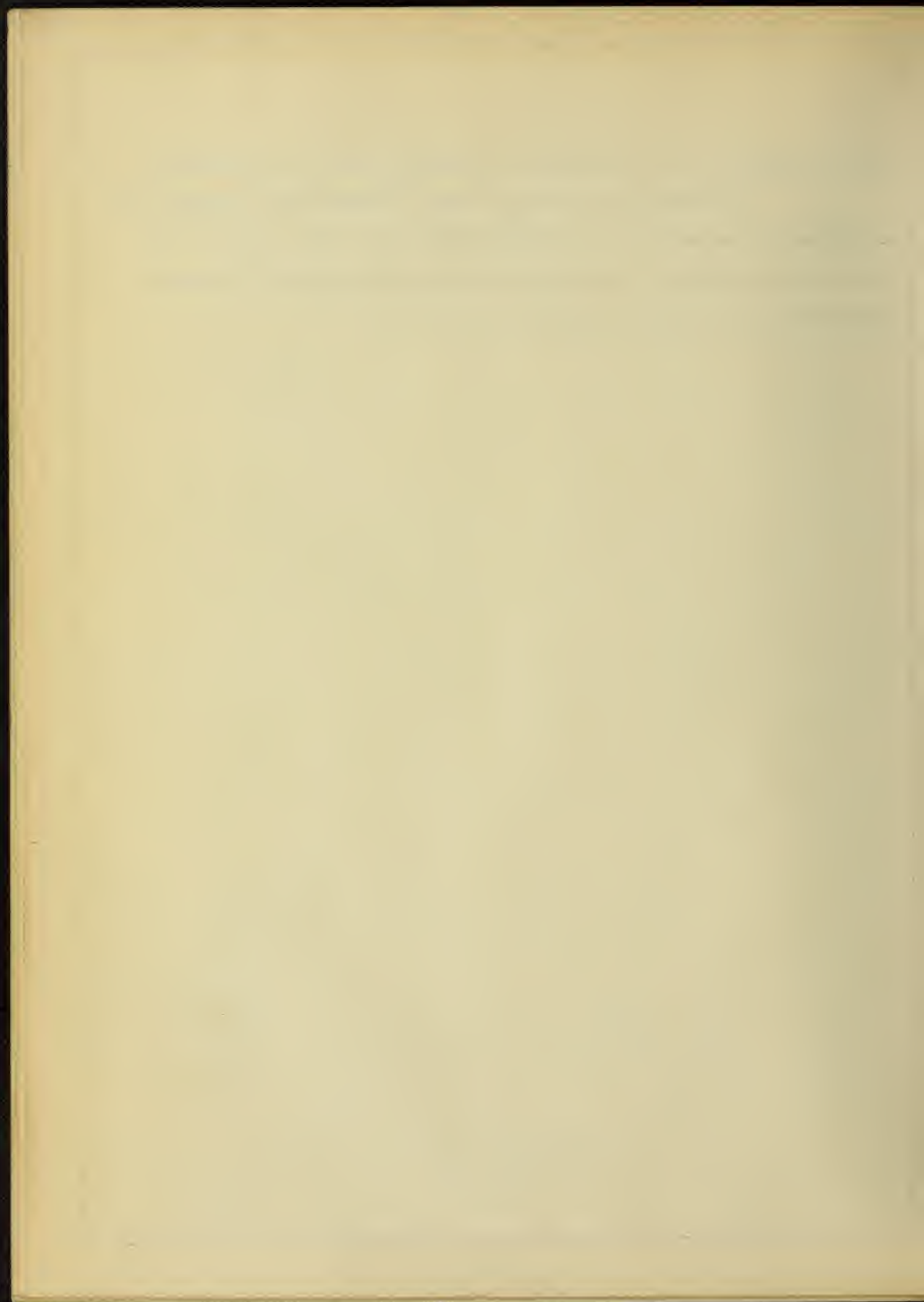
The up keep is greater for the gas light than for the carbon since new lights are furnished by the Electric Light Co. free but it is below that of the tungsten lights.

The gas light gives out more heat than either of the electric lights and has the added disadvantage of being a little



dirtier perhaps but if the gas is clean this dirt is negligible.

Therefore for office or any overhead work the gas arc is better suited and more economical than any other form of light would be providing, of course, that the gas is supplied at a rate approaching one dollar a thousand cubic feet.



The determination of the factor of absorption of the mirror.

Readings

562

560

561

565

562

564

563

560

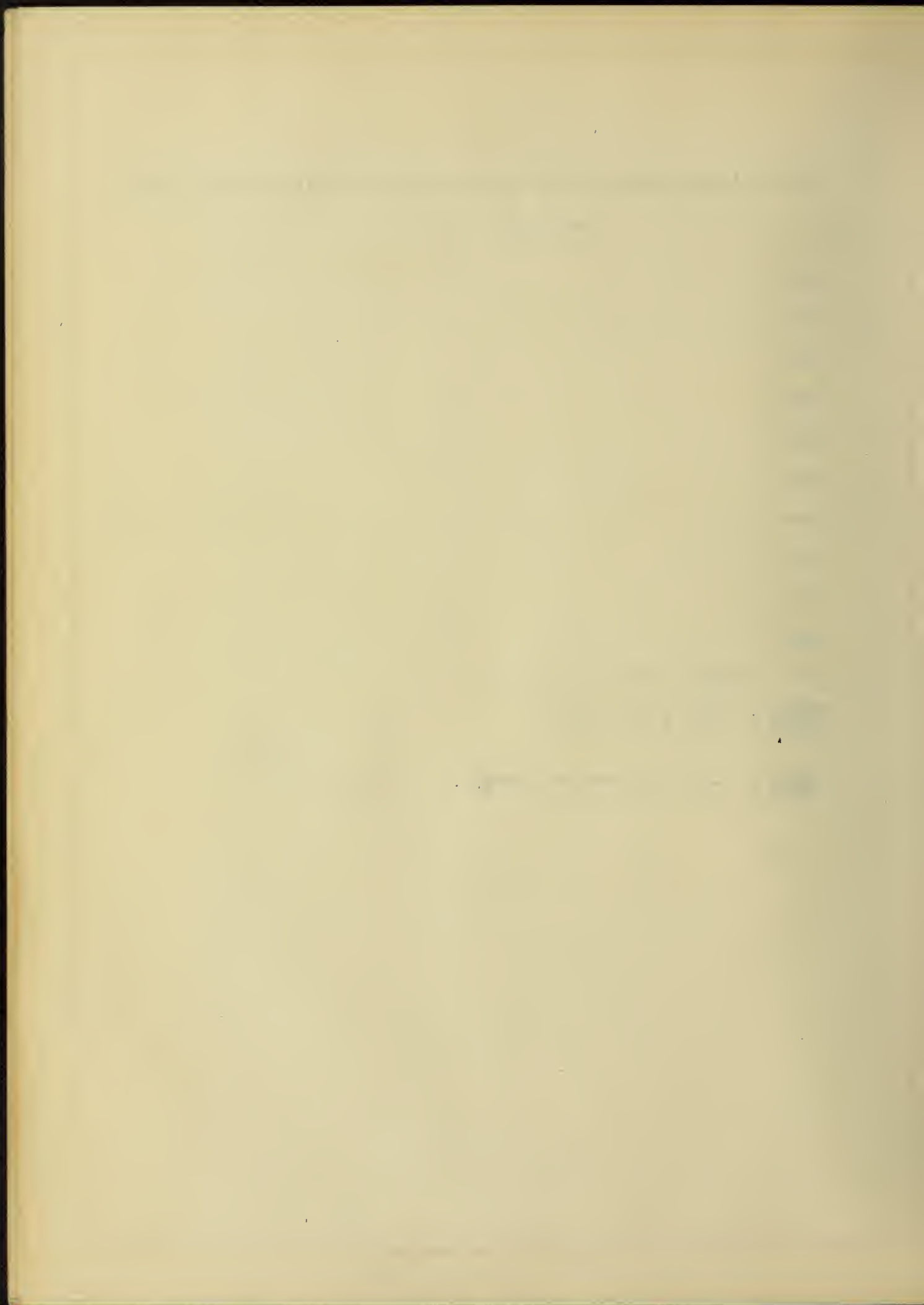
561

562

562 Average. Difference 416

$$\frac{562.2}{416} = 1.82 \times 16 = 29.3$$

$$\frac{39.0}{29.3} = 1.333 \text{ corrective factor.}$$



Sample Calculations

Candle power per cubic foot of gas per hour corrected.

Horizontal circle, vertical, thirty degrees.

$$\frac{193^2}{37249} = 37249 \quad \frac{785^2}{614225} = 614225 \quad \frac{614225}{37249} = 16.6 \text{ factor}$$

$$16.6 \times 16 = 265 \text{ C.P.} \quad \frac{265}{8.2} = 32.3 \text{ C.P. per cu.ft. of gas per hr.}$$

$$32.3 \times 1.33 = 43.0 \text{ C.P. per cu.ft. of gas per hour corrected.}$$

Mean spherical intensity.

Rousseau's diagram.

$$\text{Area of figure abcd (page)} = 10.37 \quad \text{Unit} = 47.2 \text{ C.P.}$$

$$\text{Area of figure bcd} = 5.47$$

$$\frac{\text{bcd}}{\text{abcd}} = \frac{5.47}{10.37} = .526 \times 47.2 = 24.8 \text{ mean spherical intensity.}$$

By formula

$$\frac{H}{2} + \frac{M}{4} = S \quad H = \text{average horizontal candle power}$$

$$M = \text{maximum candle power}$$

$$\frac{26}{2} + \frac{47.2}{4} = 24.4 \text{ mean spherical intensity.}$$

Spherical Reduction Factor

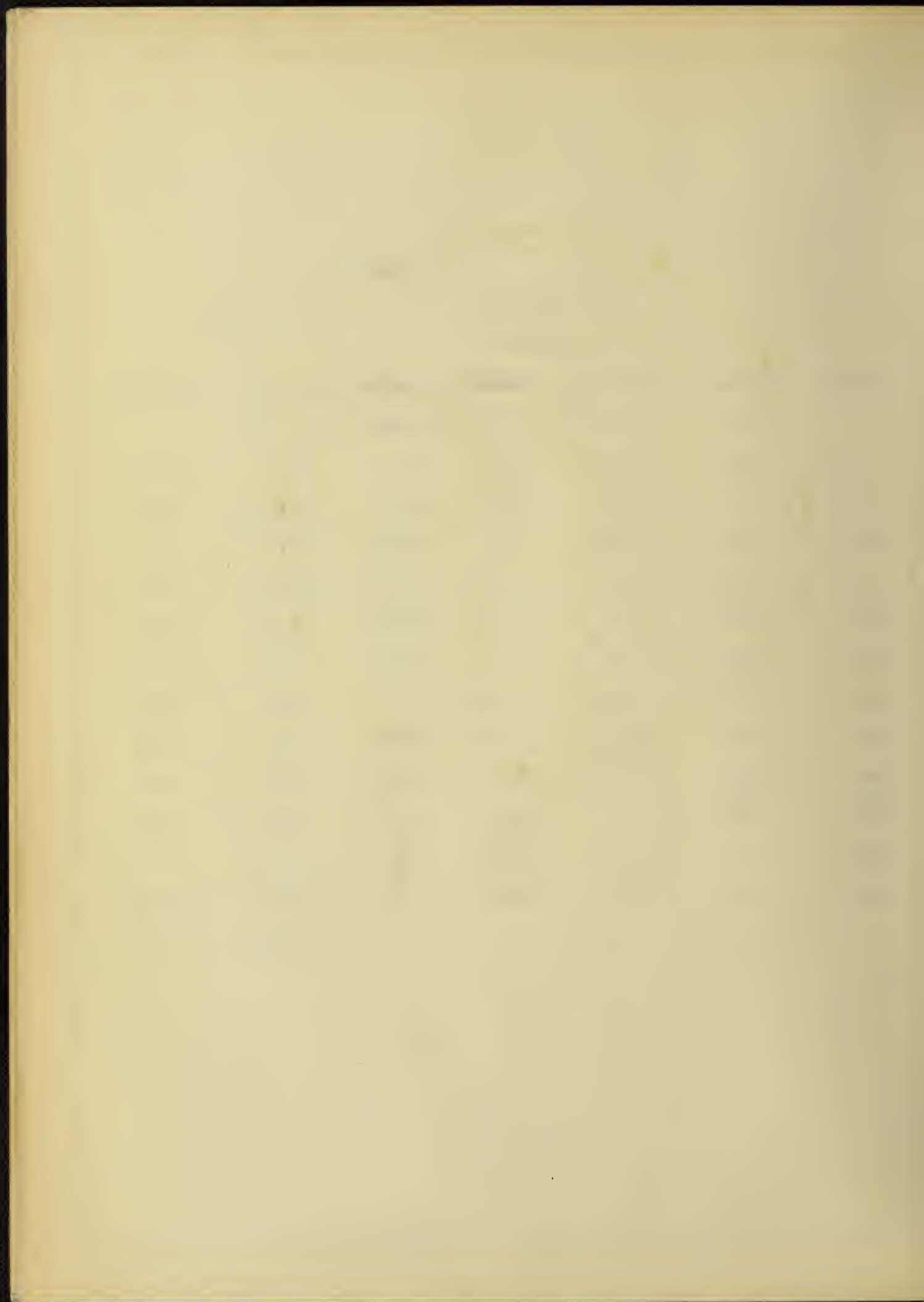
$$\frac{\text{Mean spherical candle power}}{\text{Mean Horizontal candle power}} = \frac{24.8}{26.0} = .953$$



Horizontal Circles

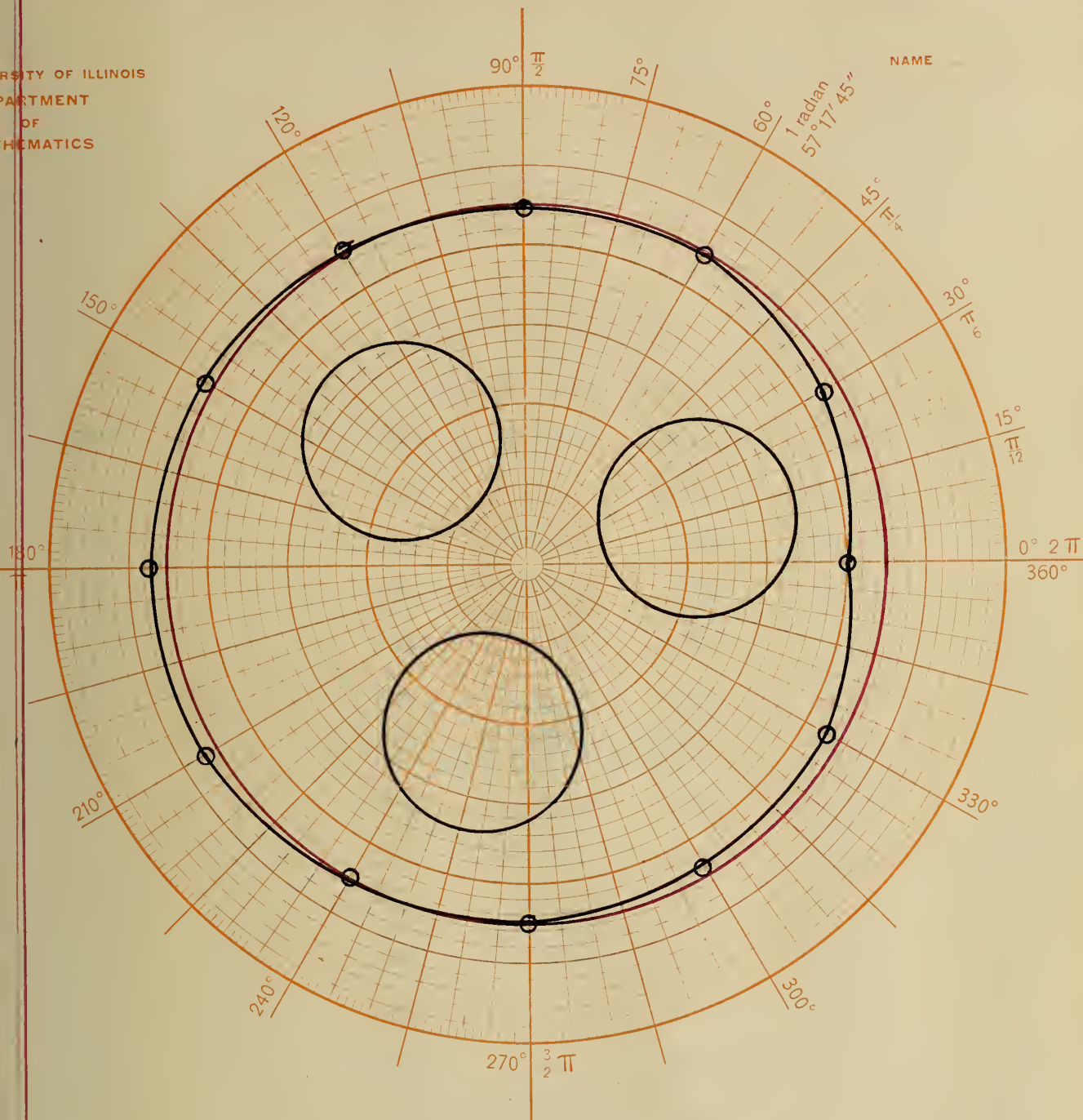
Vertical

Degree	Average	Difference	Factor	Candle	C.P.per	Corrected
	cm.	cm.		Power	cu. ft.	
0	198	780	15.1	248	30.4	40.4
30	193	785	16.6	265	32.3	43.0
60	190	788	17.2	276	33.6	44.7
90	190	788	17.2	276	33.6	44.7
120	189	789	17.8	284	34.3	45.8
150	188	790	17.9	286	34.9	46.4
180	186	792	18.1	191	35.4	47.2
210	187	791	18.0	288	35.1	46.8
240	189	789	17.8	284	34.3	45.8
270	190	788	17.2	276	33.6	44.7
300	192	786	16.8	269	32.8	43.6
330	193	785	16.6	265	32.3	<u>43.0</u>
Average -						44.8



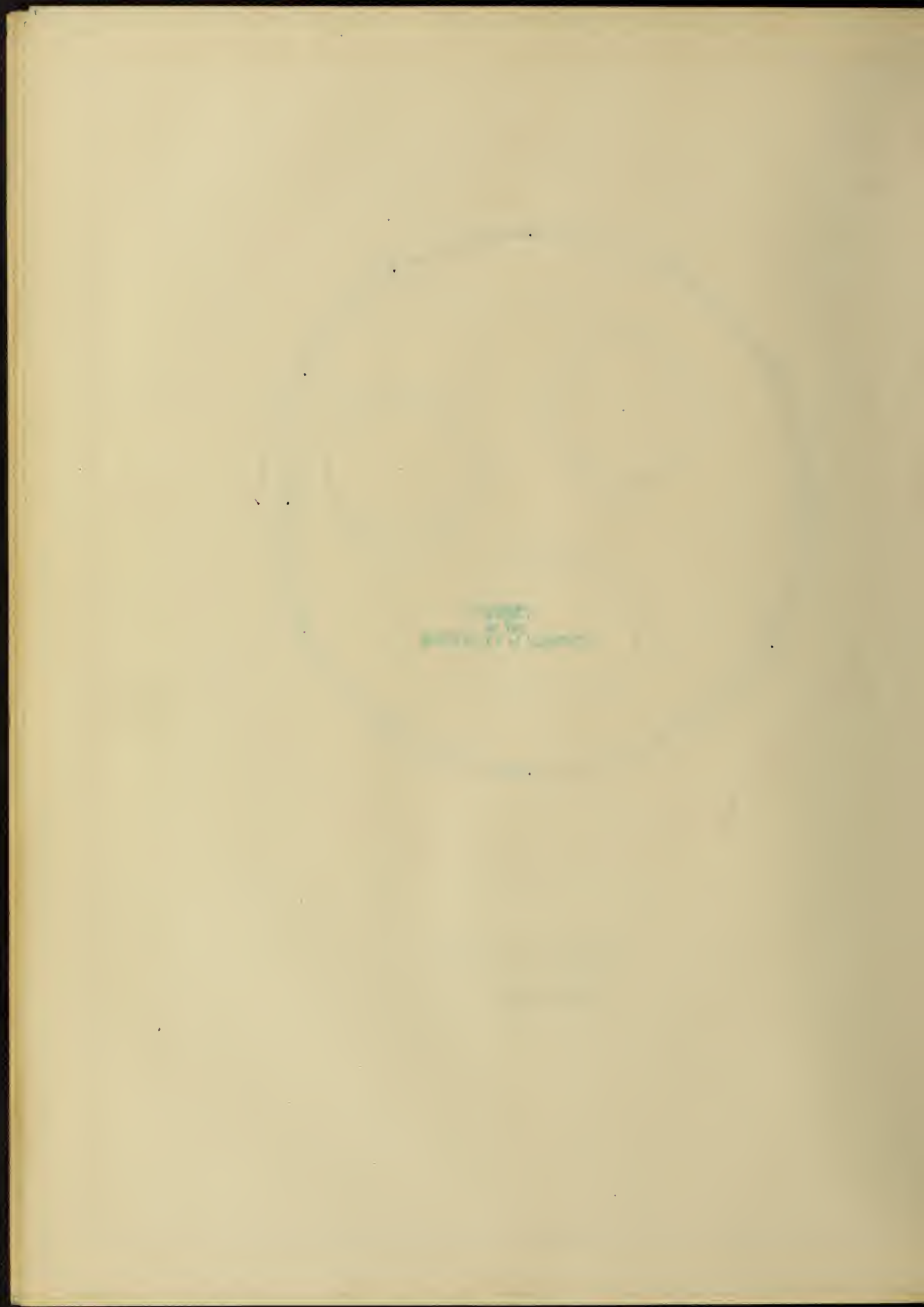
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Horizontal Circle

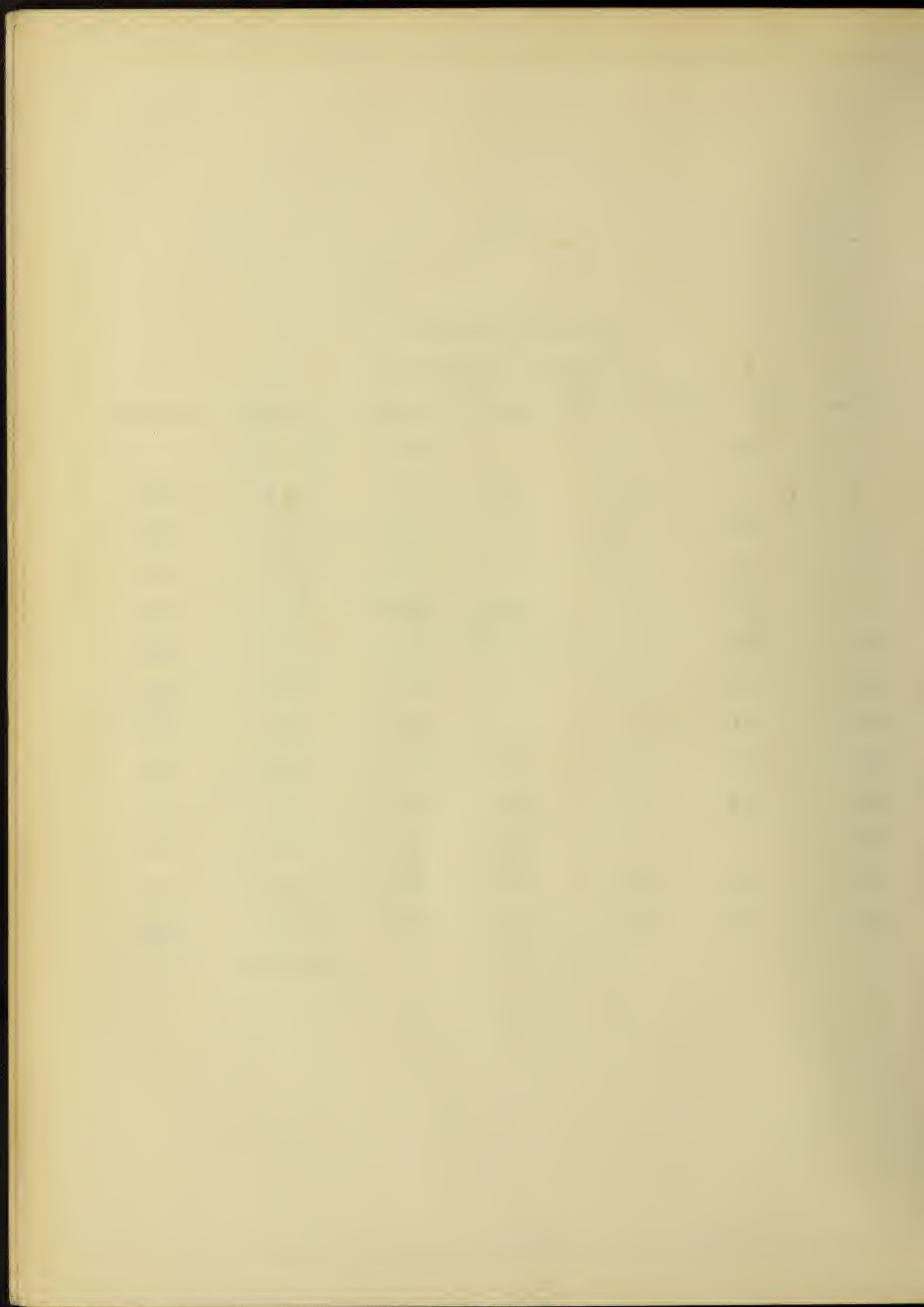
Vertical



Horizontal Circles

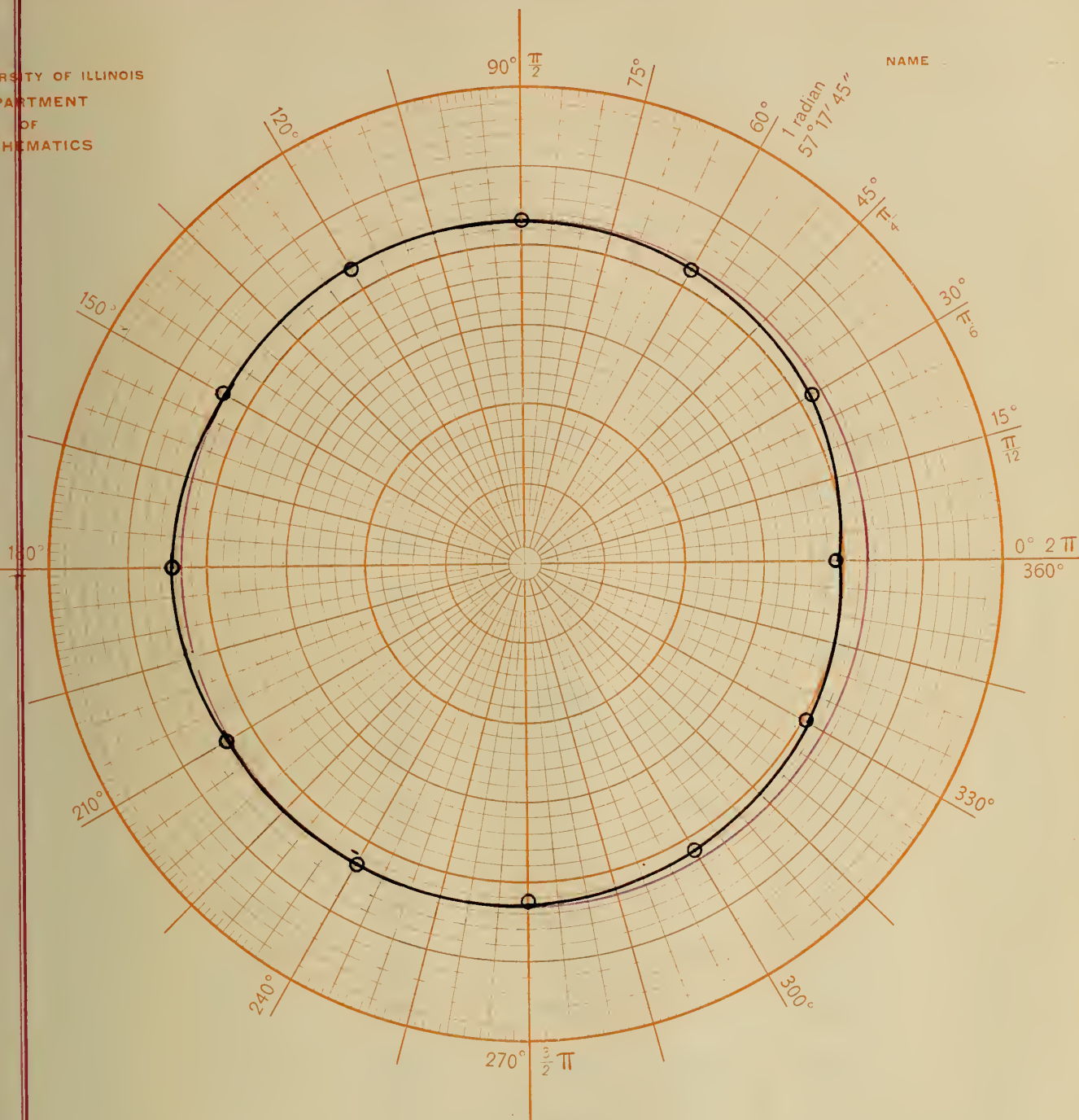
10 Degrees from Vertical

Degree	Average cm.	Difference cm.	Factor	Candle Power	C.P.per cu. ft.	Corrected
0	202	776	15.0	240	29.5	39.0
30	196	782	16.1	257	31.3	41.8
60	194	784	16.3	261	31.8	42.3
90	194	784	16.3	261	31.8	42.3
120	193	785	16.6	265	32.3	43.0
150	192	786	16.8	269	32.8	43.6
180	191	787	17.1	273	33.0	44.3
210	192	786	17.8	269	32.8	43.6
240	193	785	16.6	265	32.3	43.0
270	194	784	16.3	261	31.8	42.3
300	195	783	16.1	257	31.3	41.8
330	197	781	15.6	248	31.4	<u>40.4</u>
Average -						42.9



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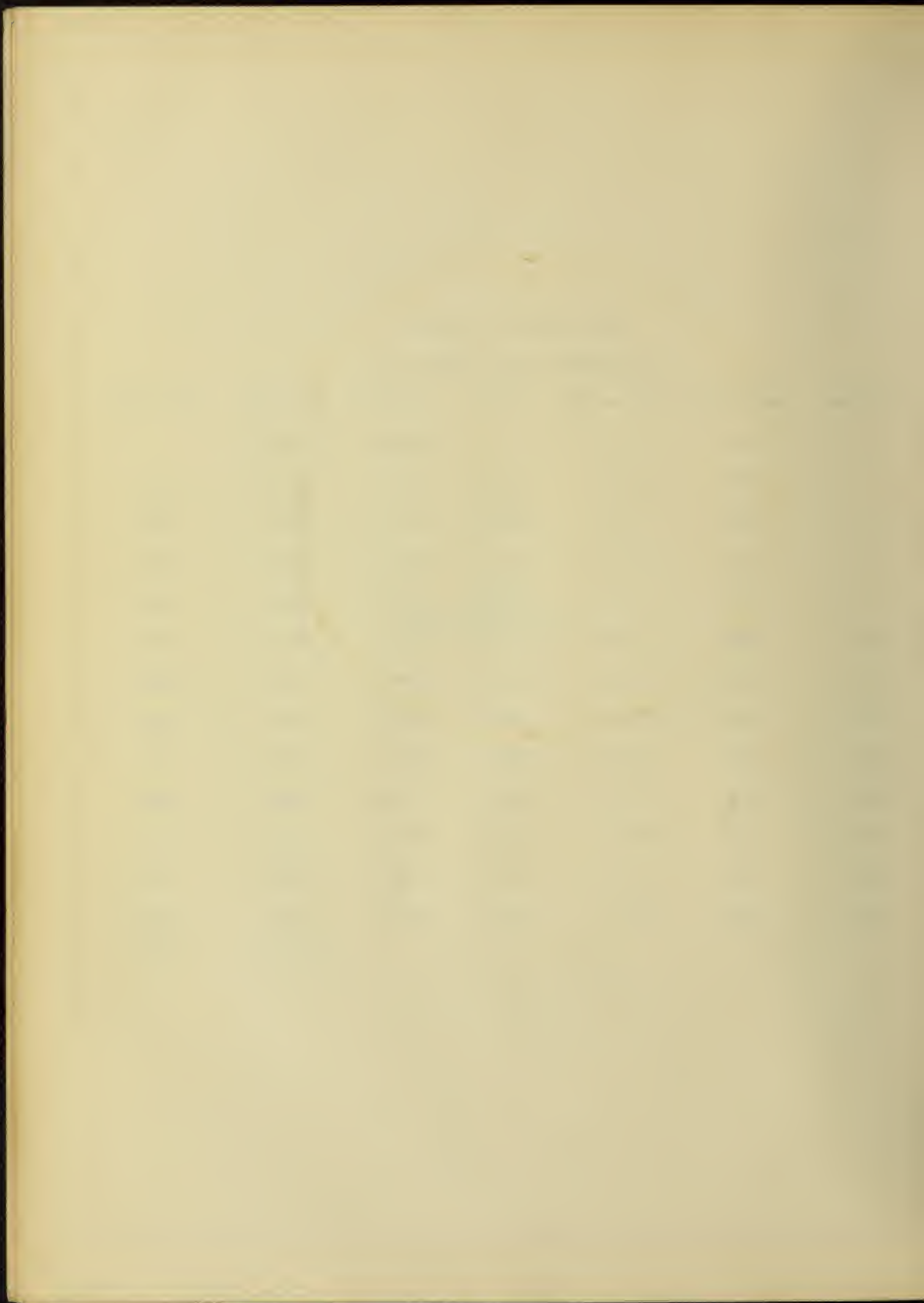
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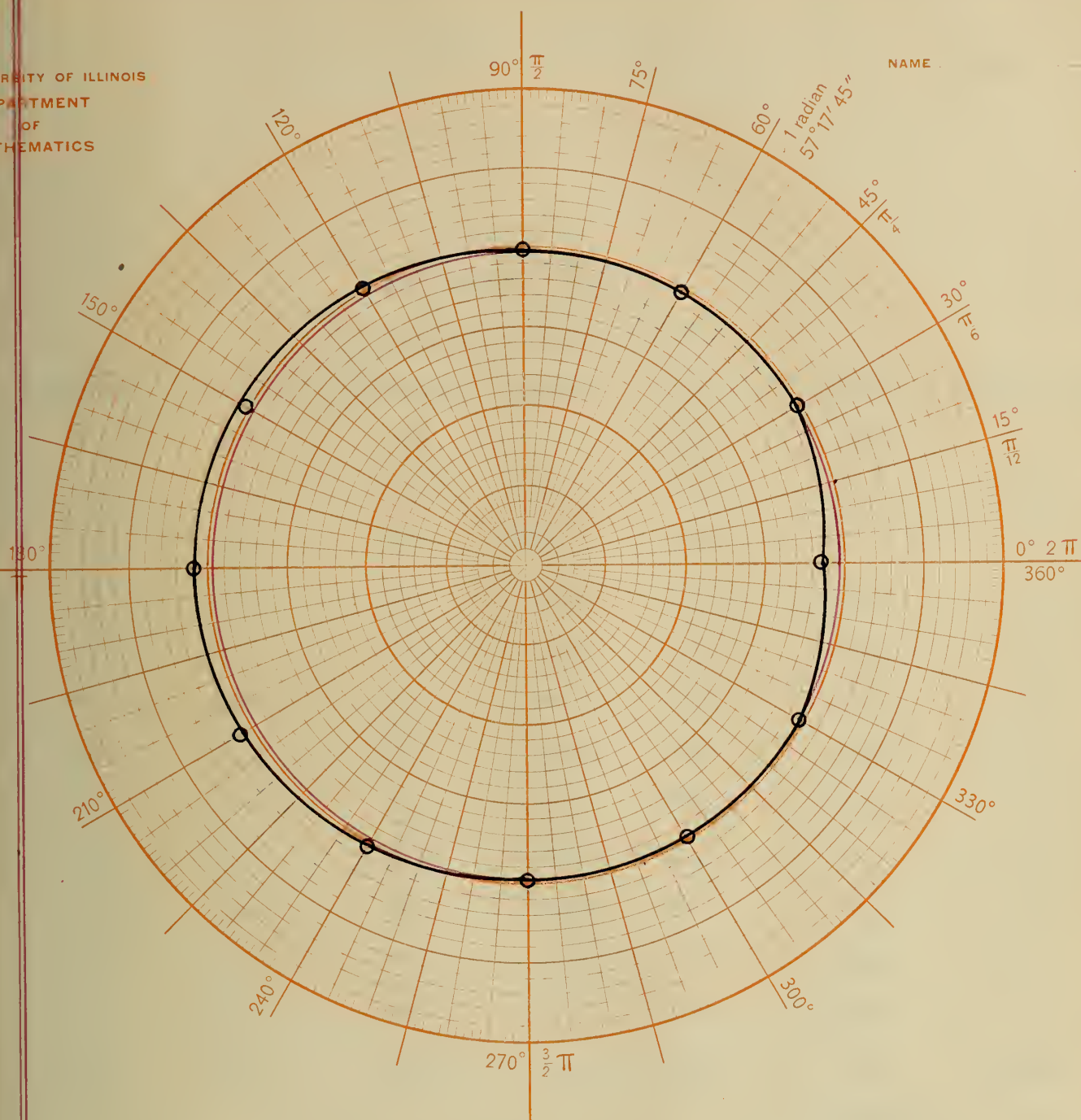


Horizontal Circles

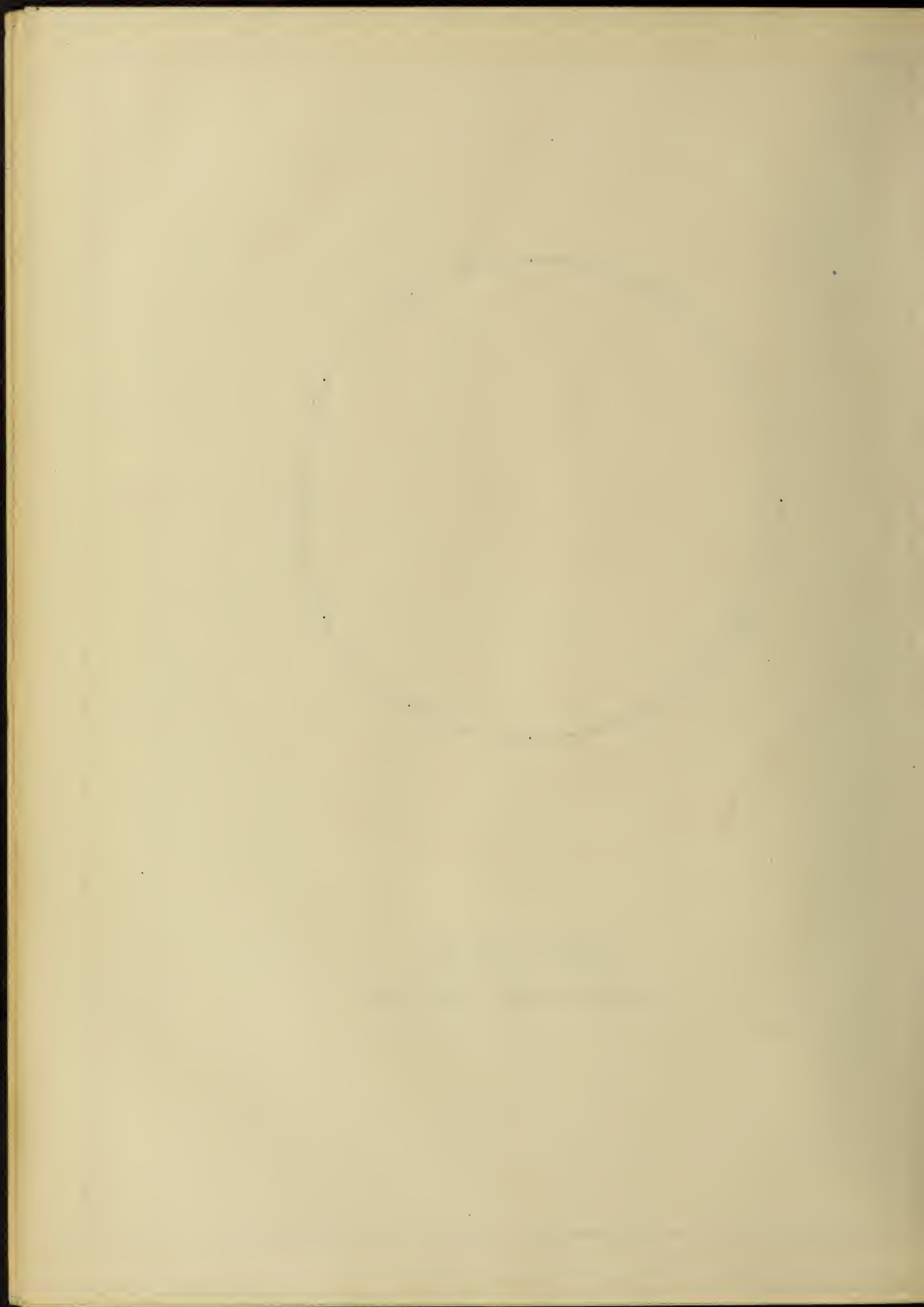
20 Degrees from Vertical

Degree	Average	Difference	Factor	Candle	C.P.per	Corrected
	cm.	cm.		Power	cu.ft.	
0	205	773	14.2	228	27.8	37.0
30	201	777	15.2	241	29.7	39.3
60	201	777	15.2	241	29.7	39.3
90	200	778	15.3	243	29.3	39.5
120	198	780	15.6	248	30.4	40.4
150	197	781	15.6	248	30.4	40.4
180	195	783	16.1	257	31.3	41.8
210	196	782	16.1	257	31.3	41.8
240	198	780	15.6	248	30.4	40.4
270	200	778	15.3	243	29.9	39.5
300	200	778	15.3	243	29.9	39.5
330	201	777	15.2	241	29.7	<u>39.3</u>
Average -						39.3





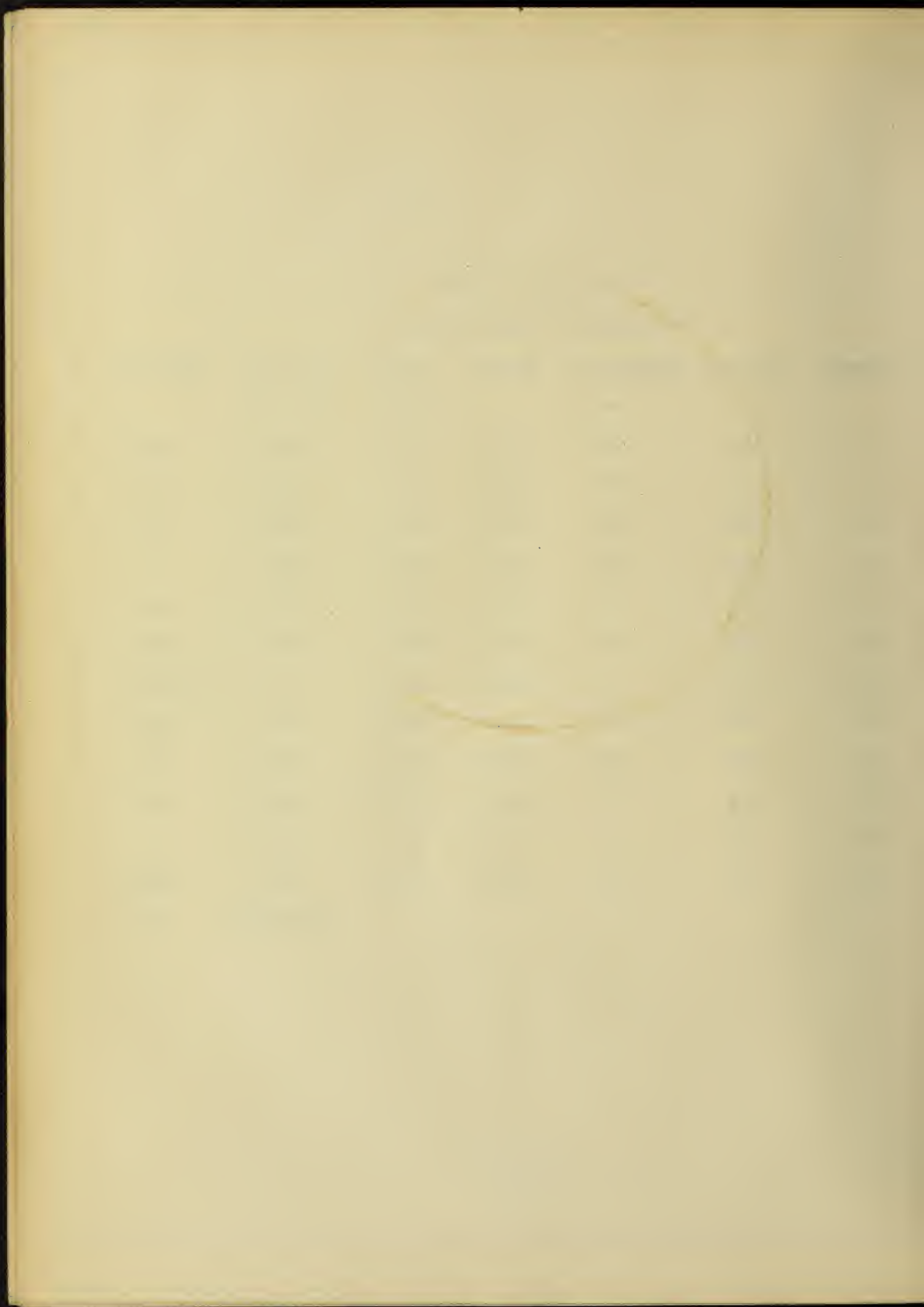
Horizontal Circle
20 Degrees from Vertical



Horizontal Circle

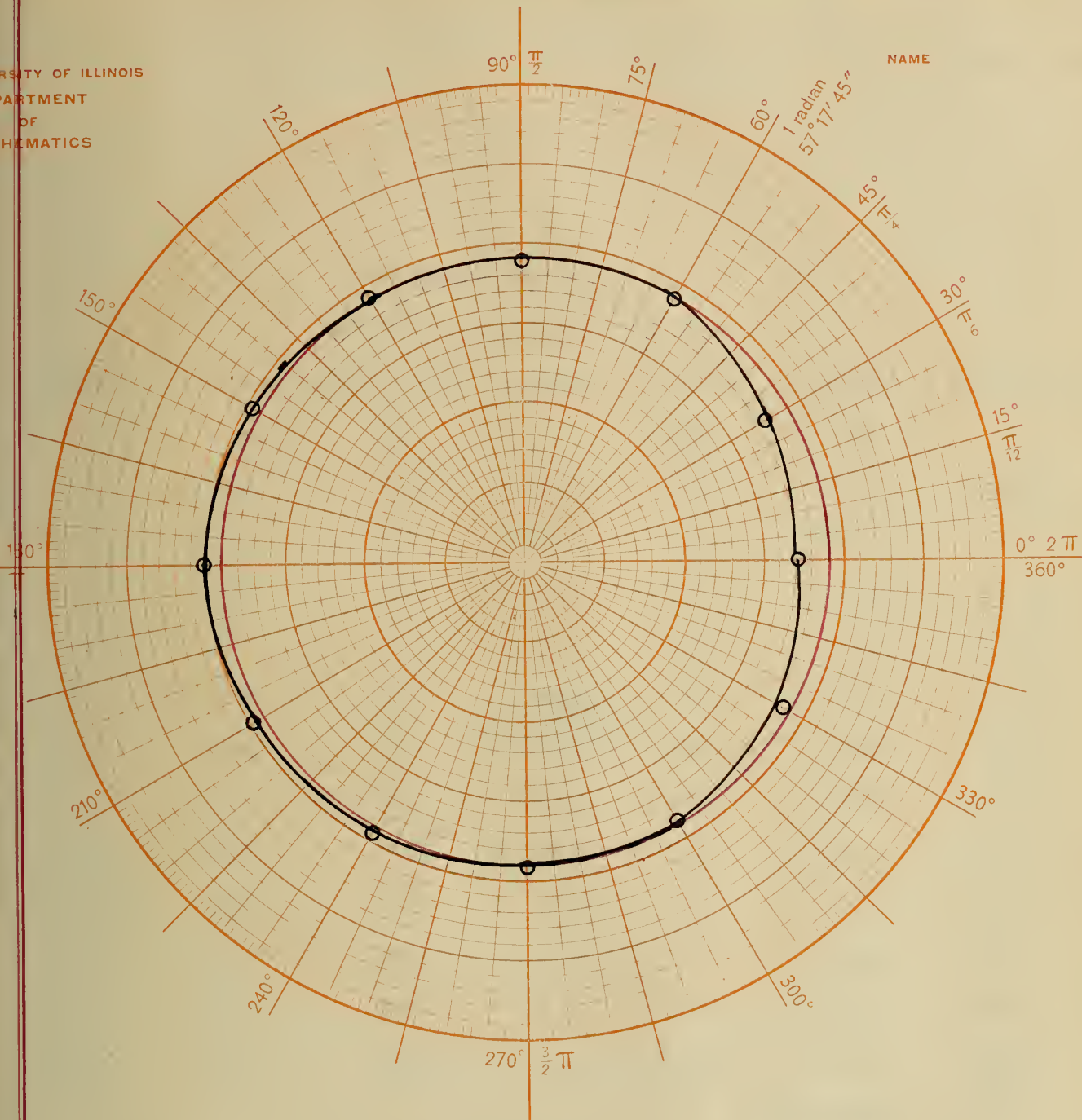
30 Degrees from Vertical

Degree	Average	Difference	Factor	Candle	C.P.per	Corrected
	cm.	cm.		Power	cu. ft.	
0	211	761	13.2	212	25.7	34.3
30	205	772	14.2	228	27.8	37.0
60	204	774	14.4	231	28.2	37.6
90	204	774	14.4	231	28.2	37.6
120	202	775	14.6	235	28.5	38.3
150	201	777	15.2	241	29.7	39.3
180	199	779	15.6	248	30.4	40.4
210	200	778	15.0	243	29.9	39.5
240	202	776	15.0	240	29.5	39.0
270	203	775	14.6	235	28.5	38.3
300	204	774	14.4	231	28.2	37.6
330	205	773	14.2	228	27.8	<u>37.0</u>
Average -						39.1



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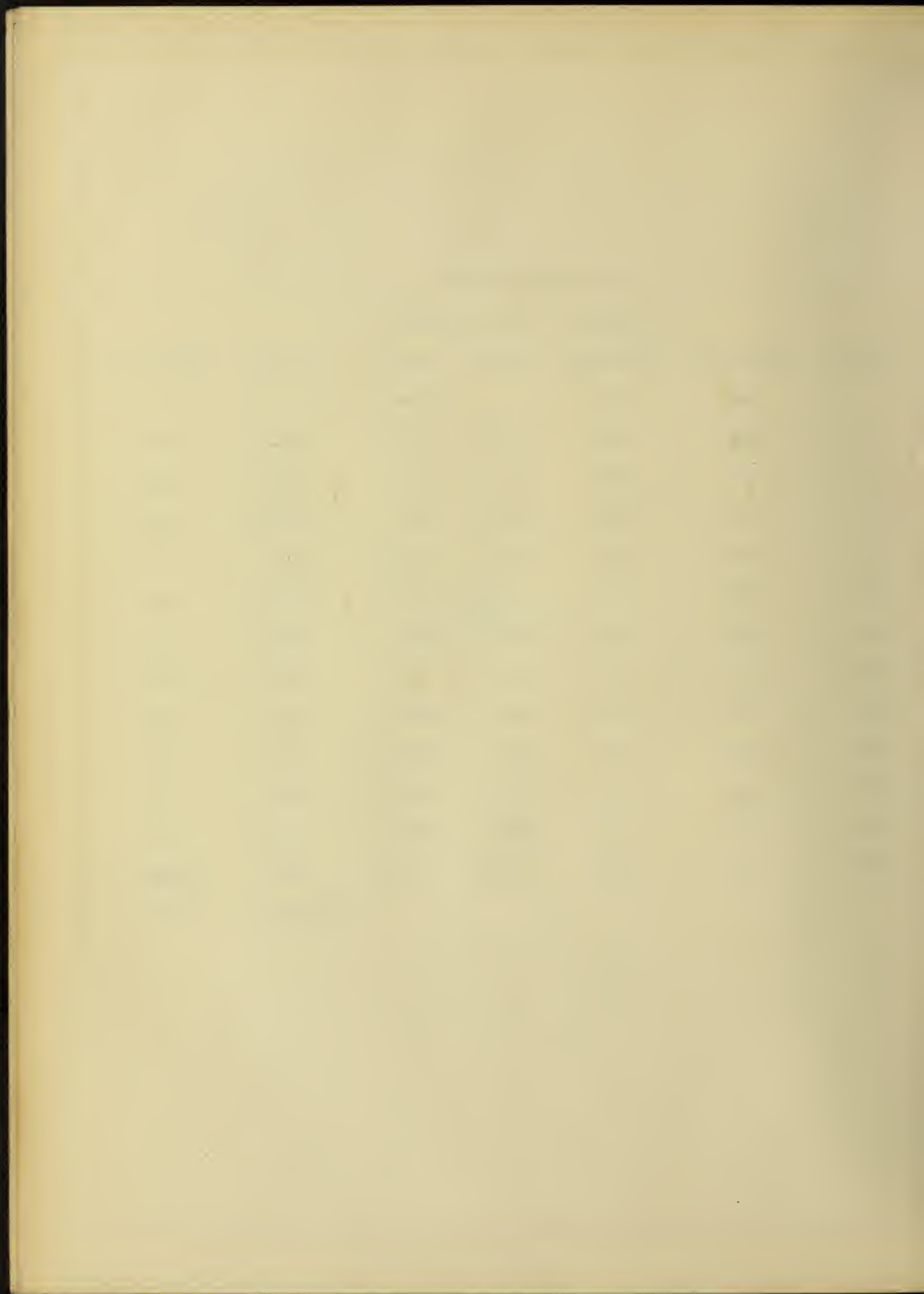
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30 Degrees from Vertical



Horizontal Circle

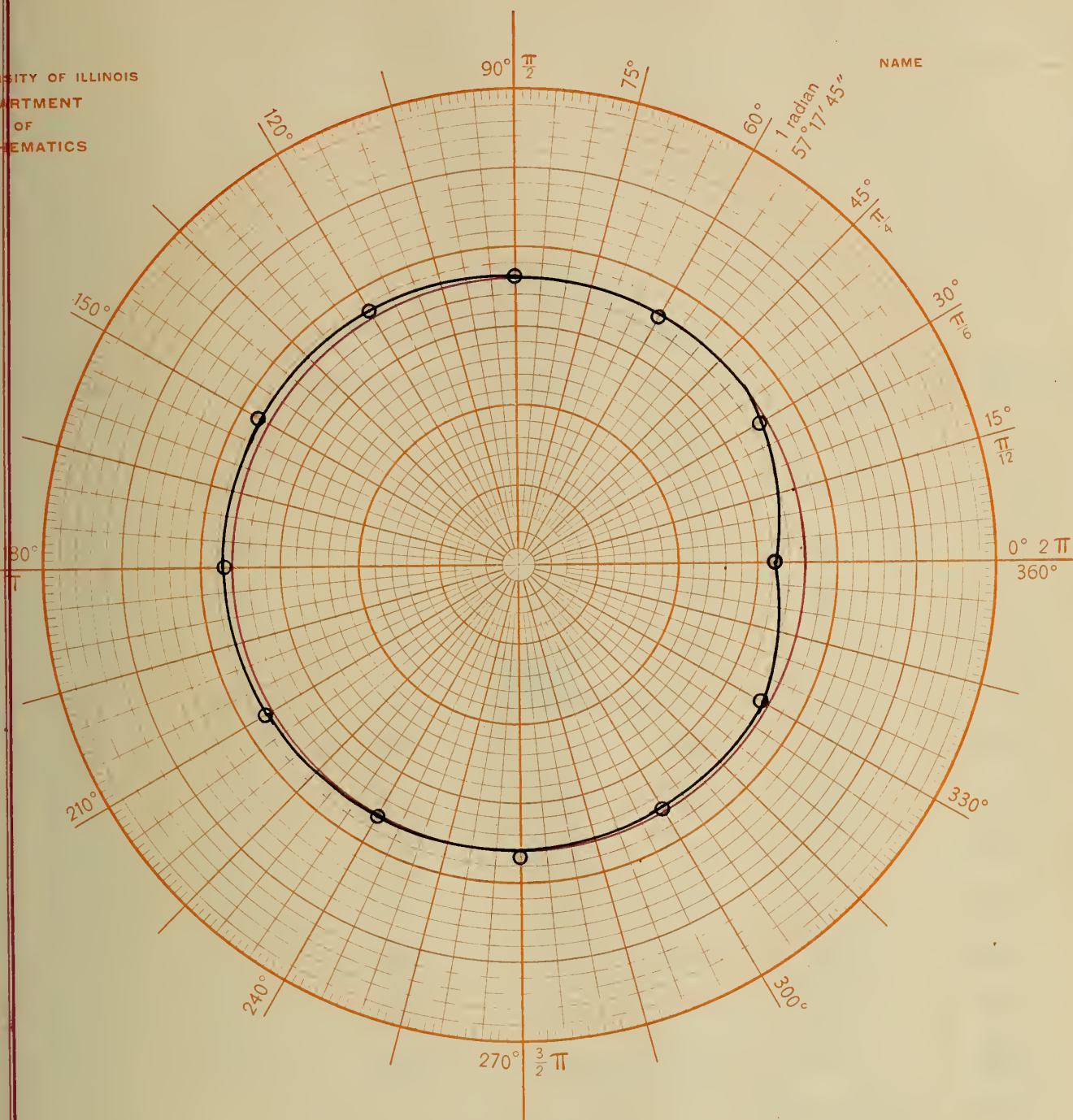
40 Degrees from Vertical

Degree	Average	Difference	Factor	Candle	C.P.per	Corrected
	cm.	cm.		Power	cu. ft.	
0	214	764	12.5	199	24.3	32.3
30	210	768	13.4	214	26.2	34.8
60	209	769	13.6	217	26.5	35.6
90	207	771	13.9	222	27.2	36.2
120	206	772	14.2	228	27.8	37.0
150	204	774	14.4	231	28.2	37.6
180	205	773	14.2	228	27.8	37.0
210	206	772	14.2	228	27.8	37.0
240	207	771	13.9	222	27.2	36.2
270	208	770	13.8	220	26.9	36.9
300	209	769	13.6	217	26.5	35.6
330	210	768	13.4	214	26.2	<u>34.8</u>
Average-						36.0



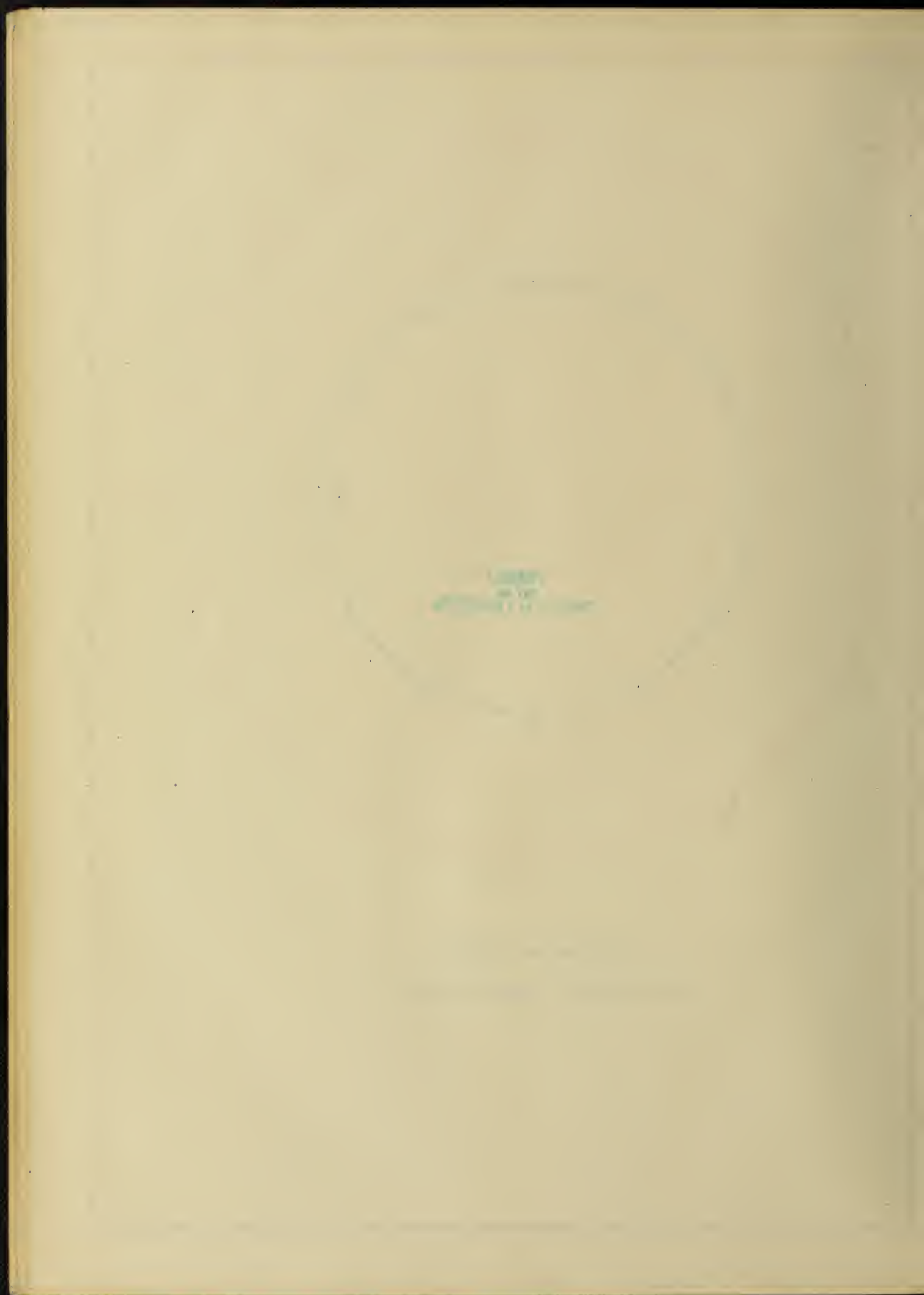
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Horizontal Circle

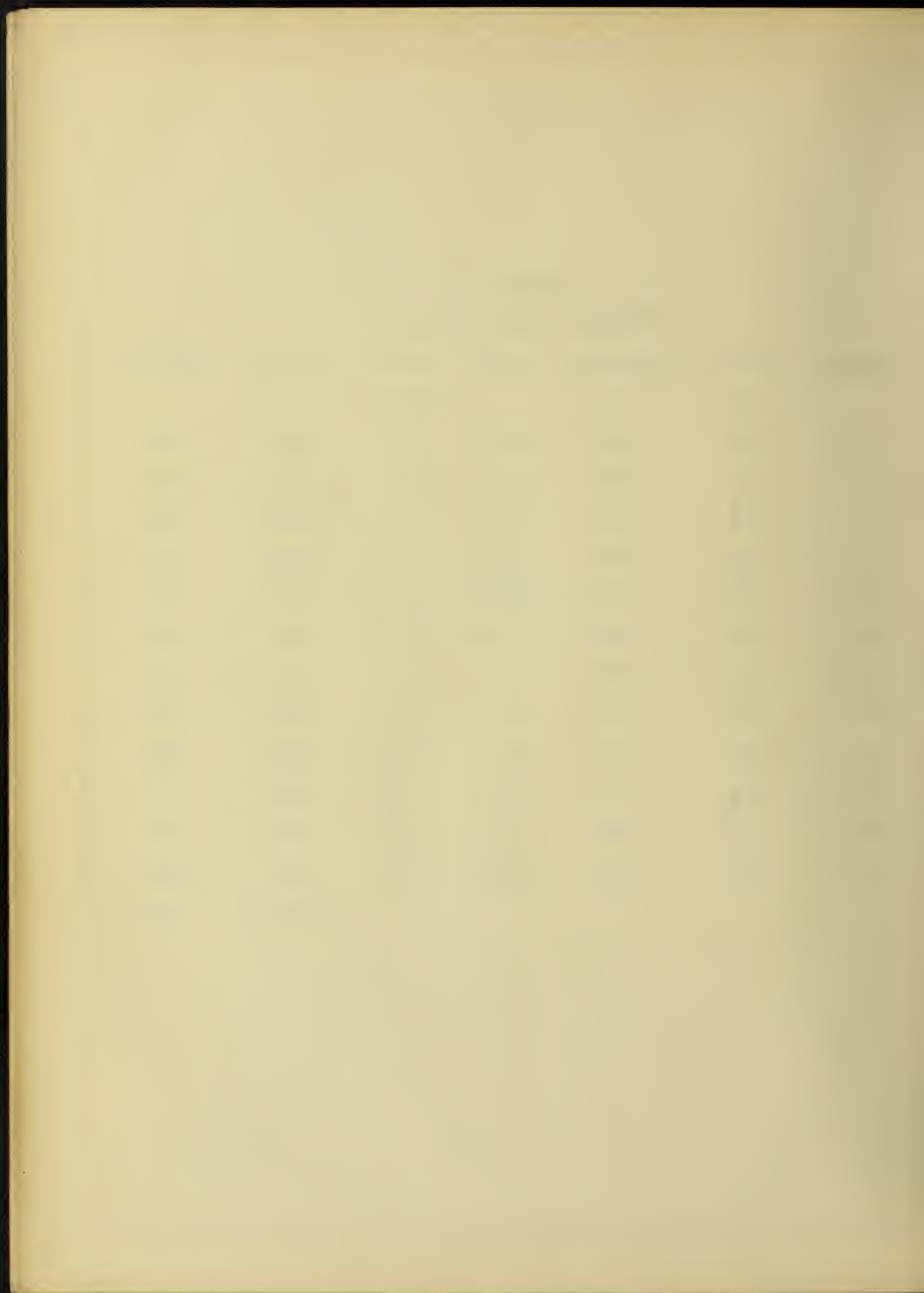
40 Degrees from Vertical



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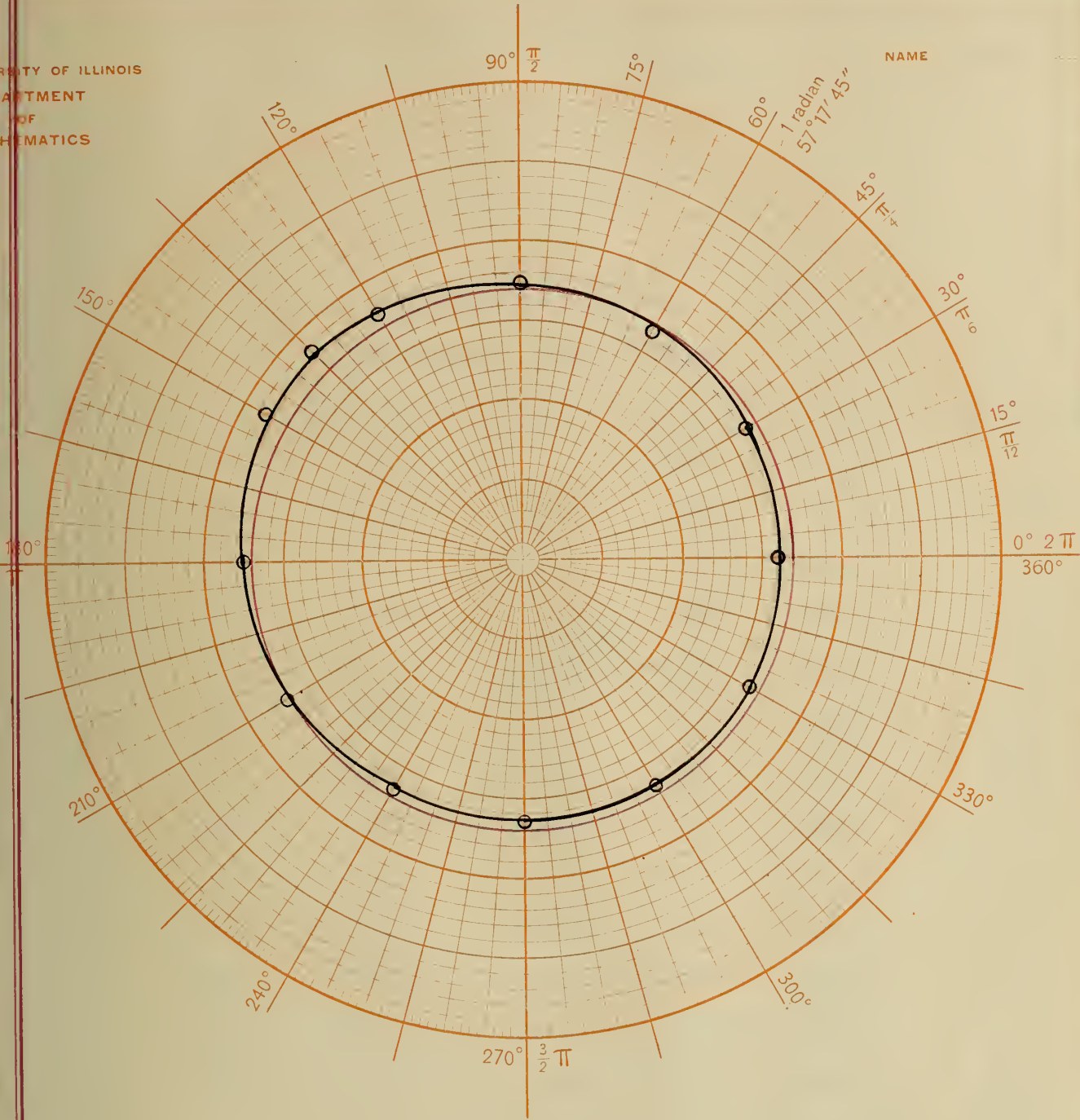
50 Degrees from Vertical

Degree	Average	Difference	Factor	Candle	C.P.per	Corrected
	cm.	cm.		Power	cu. ft.	
0	215	763	12.3	197	24.0	32.0
30	214	764	12.5	199	24.3	32.3
60	212	766	12.7	203	24.8	33.1
90	210	768	13.4	214	26.2	34.8
120	209	769	16.6	217	26.5	35.6
150	208	770	13.8	220	26.9	36.9
180	210	768	13.4	214	26.2	34.5
210	211	767	13.2	212	25.7	34.3
240	212	766	12.7	203	24.8	33.1
270	212	766	12.7	203	24.8	33.1
300	213	765	12.6	201	24.5	32.7
330	213	765	12.6	201	24.5	<u>32.7</u>
Average -						33.8



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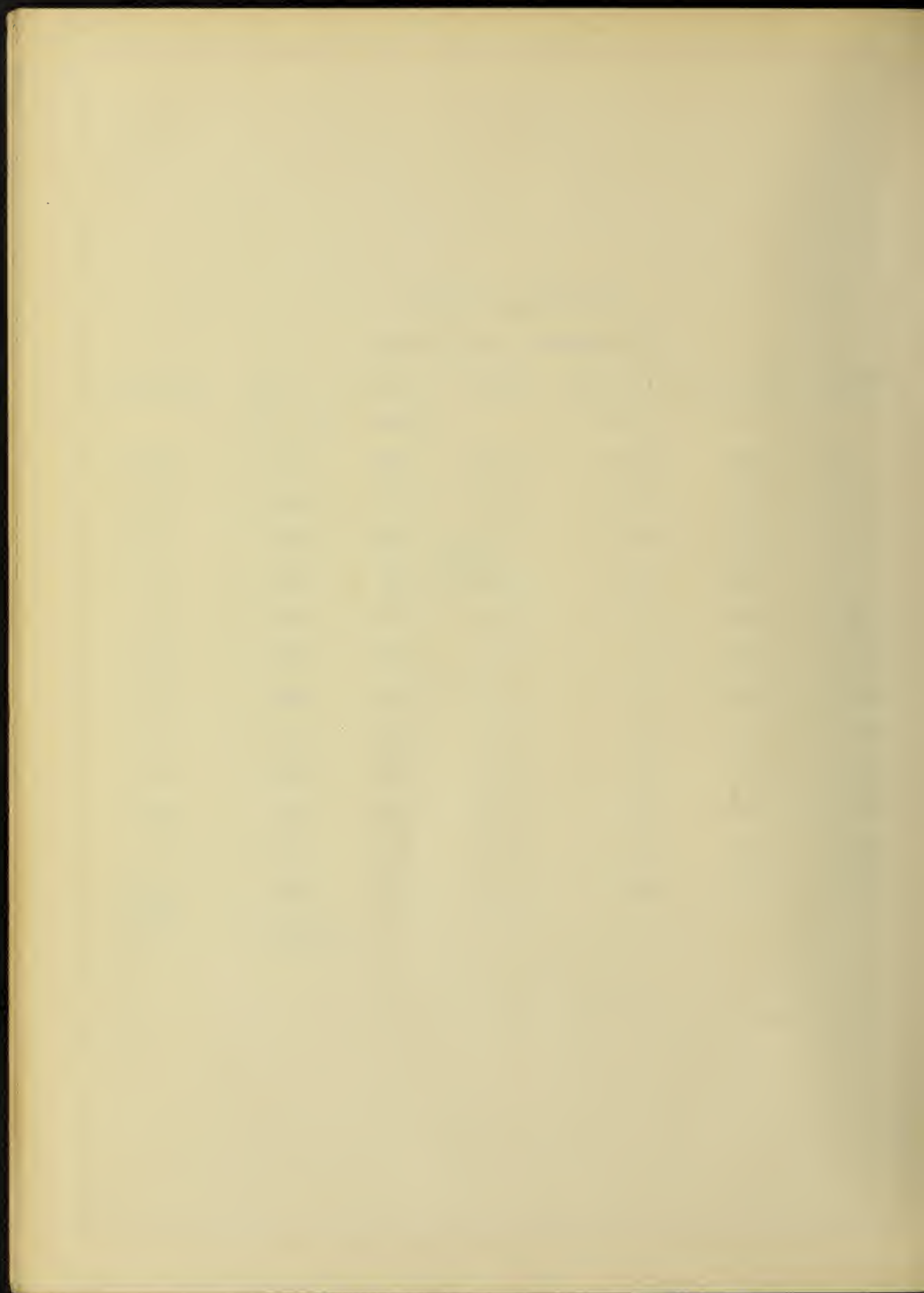
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Horizontal Circles

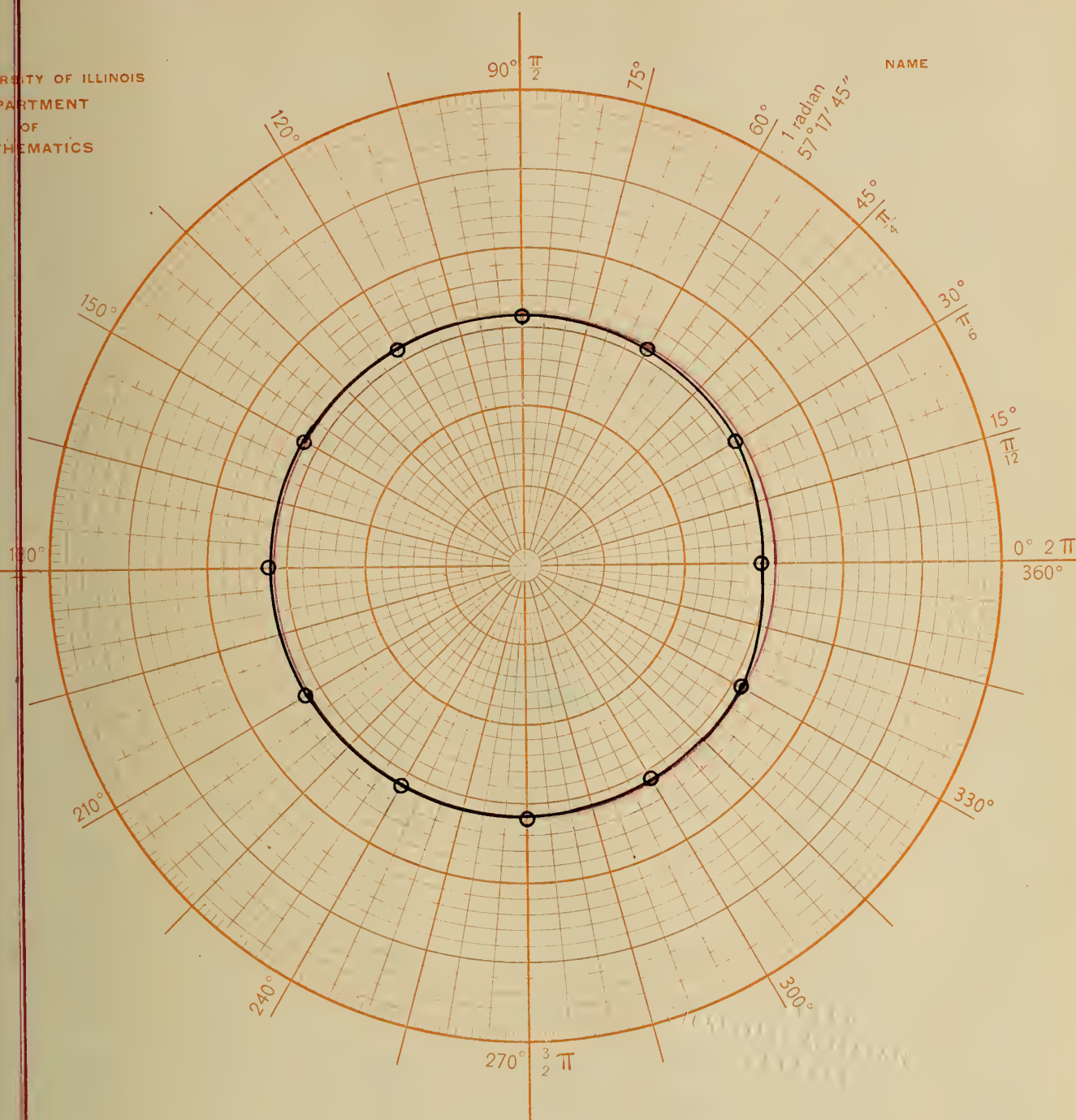
60 Degrees from Vertical

Degree	Average	Difference	Factor	Candle	C.P. per	Corrected
	cm.	cm.		Power	cu. ft.	
0	224	754	11.4	182	22.2	29.5
30	220	758	11.9	191	23.2	30.8
60	219	759	12.0	194	23.5	31.2
90	218	760	12.2	197	23.8	31.7
120	218	760	12.2	197	23.8	31.7
150	217	761	12.3	198	24.0	32.0
180	216	762	12.5	200	24.3	32.3
210	216	762	12.5	200	24.3	32.3
240	217	761	12.3	198	24.0	32.0
270	217	761	12.3	198	24.0	32.0
300	219	759	12.0	194	23.5	31.2
330	219	759	12.0	194	23.5	<u>31.2</u>
Average -						31.4



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Horizontal Circle
60 Degrees from Vertical

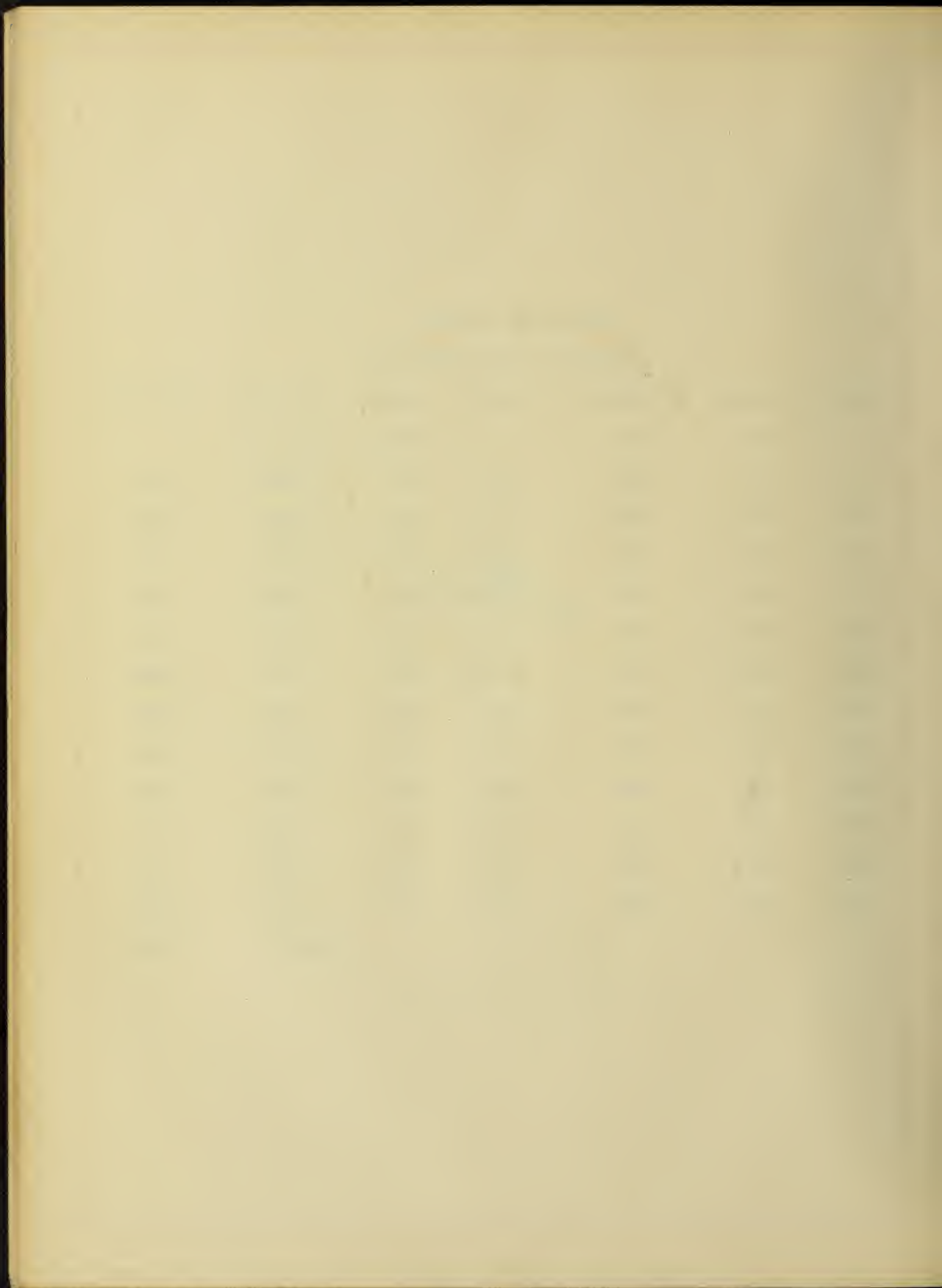


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Horizontal Circle

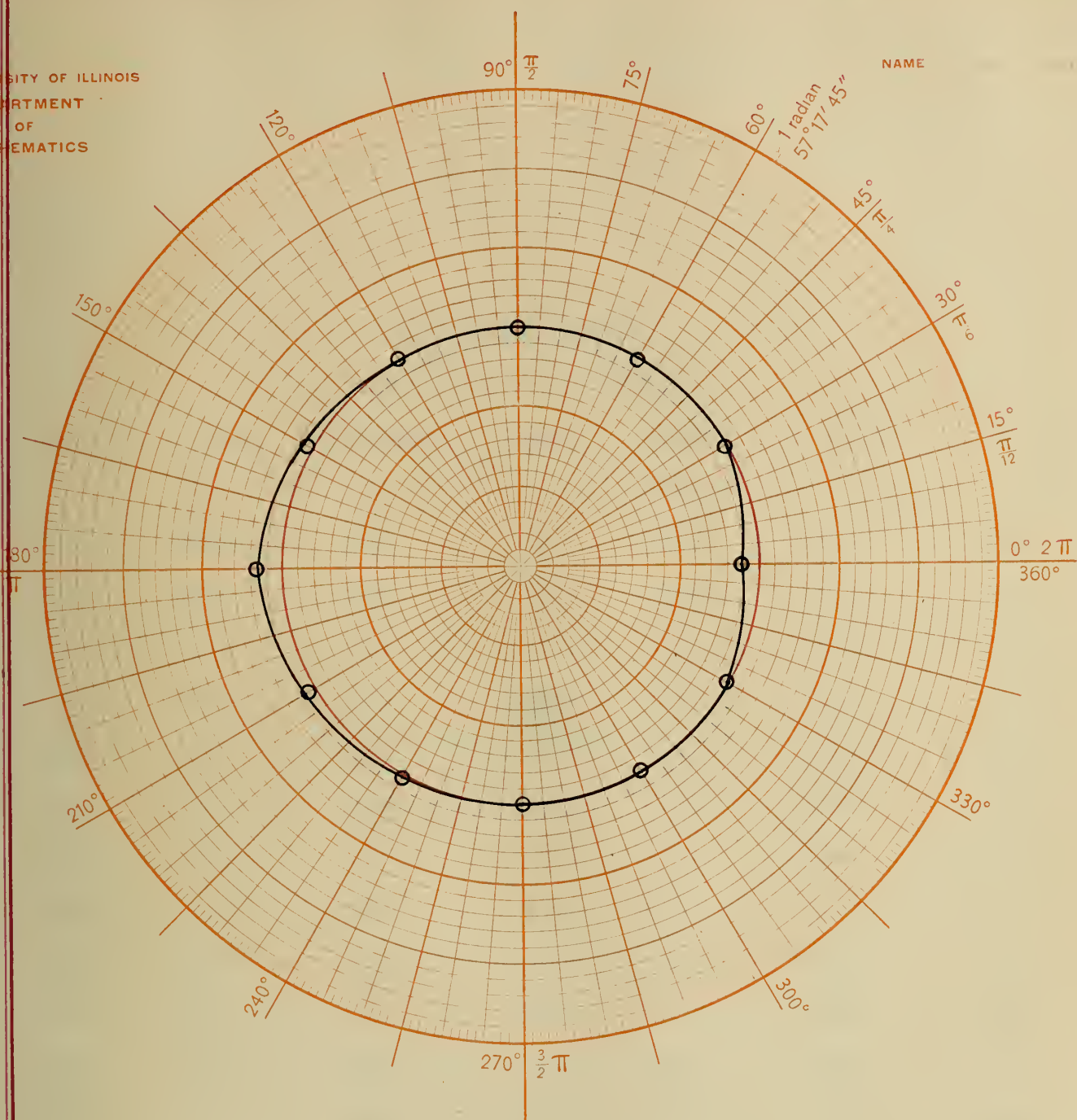
70 Degrees from Vertical

Degree	Average	Difference	Factor	Candle	C.P.per	Corrected
	cm.	cm.		Power	cu. ft.	
0	229	749	10.7	171	20.8	27.7
30	224	754	11.4	182	22.2	29.5
60	224	754	11.4	182	22.2	29.5
90	223	755	11.5	184	22.4	29.8
120	222	756	11.6	186	22.7	30.2
150	220	758	11.9	191	23.2	30.8
180	219	759	12.0	194	23.5	31.2
210	220	758	11.8	191	23.2	30.8
240	221	757	11.8	188	22.9	30.5
270	223	755	11.5	184	22.4	29.8
300	223	755	11.5	184	22.4	29.8
330	224	754	11.4	182	22.2	<u>29.5</u>
Average-						30.0



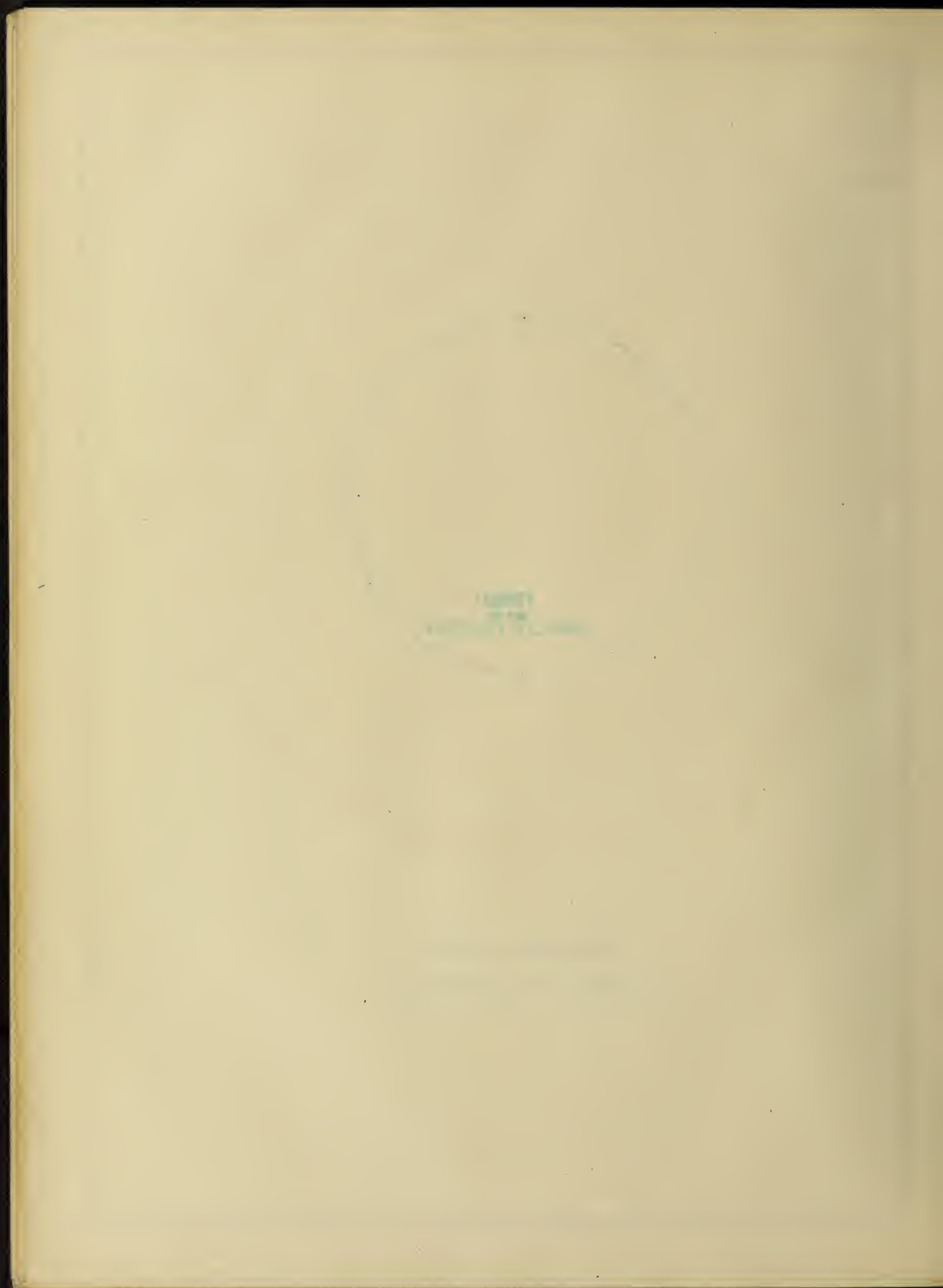
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Horizontal Circle

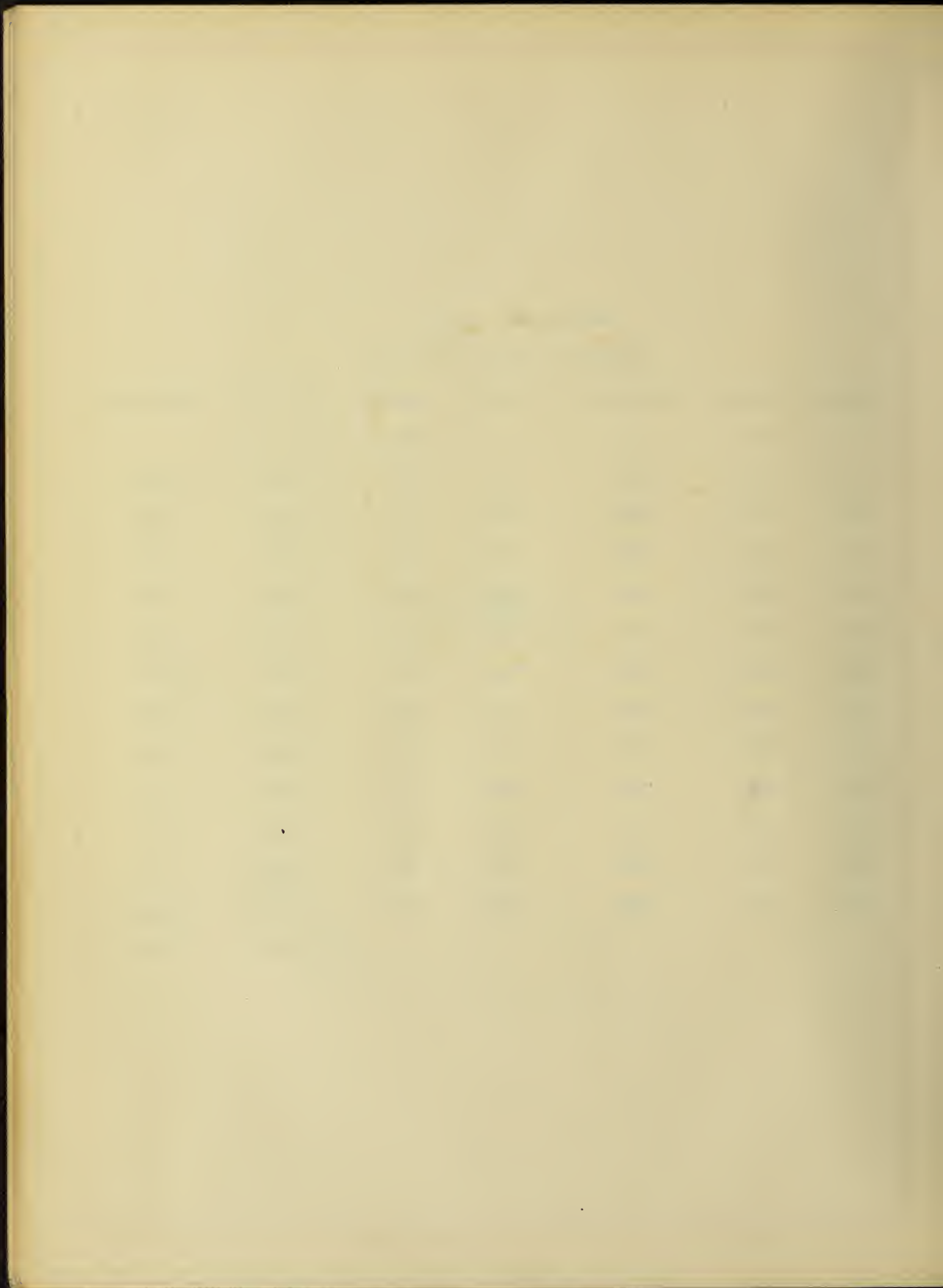
70 Degrees from Vertical



Horizontal Circle

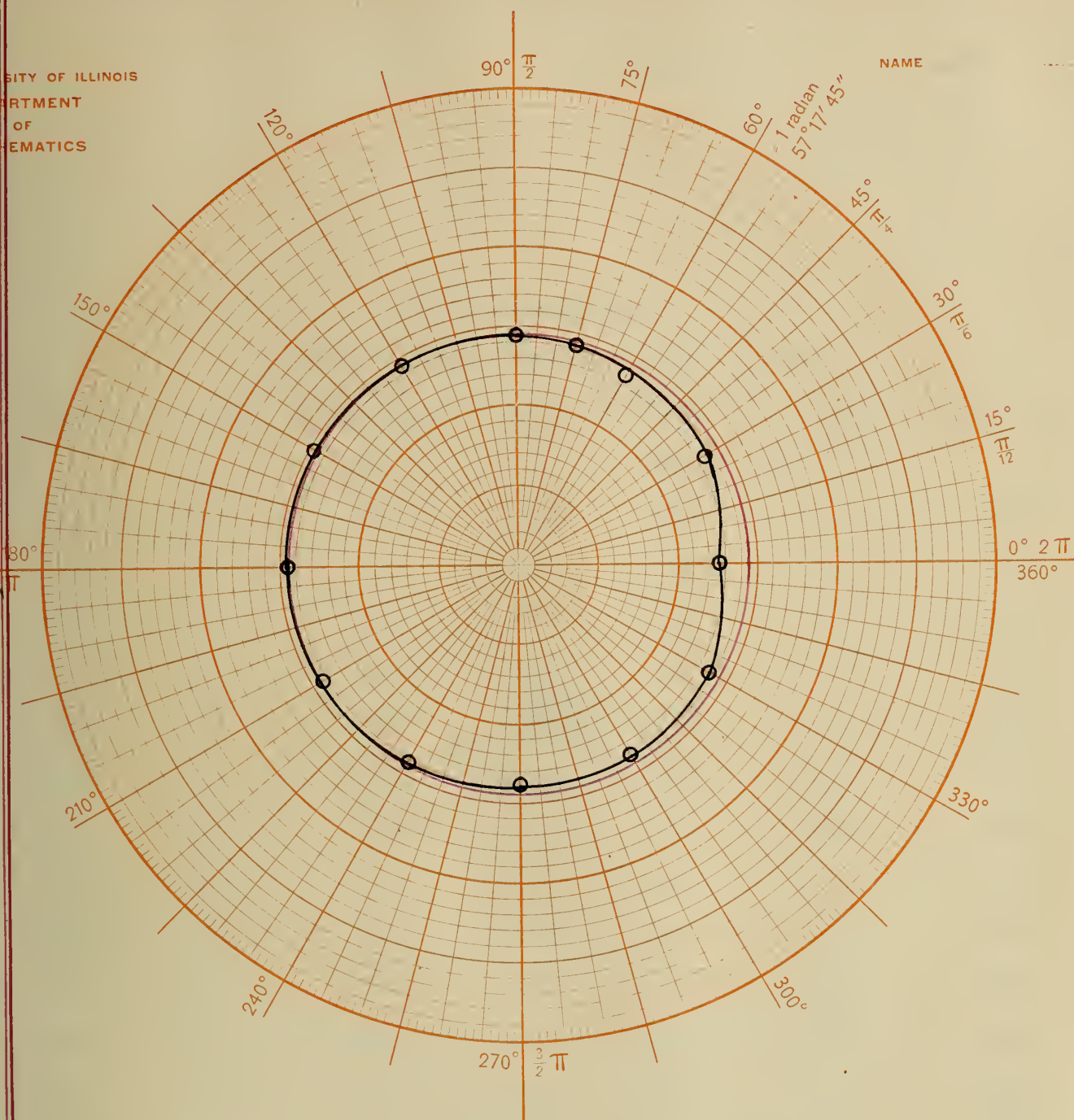
80 Degrees from Vertical

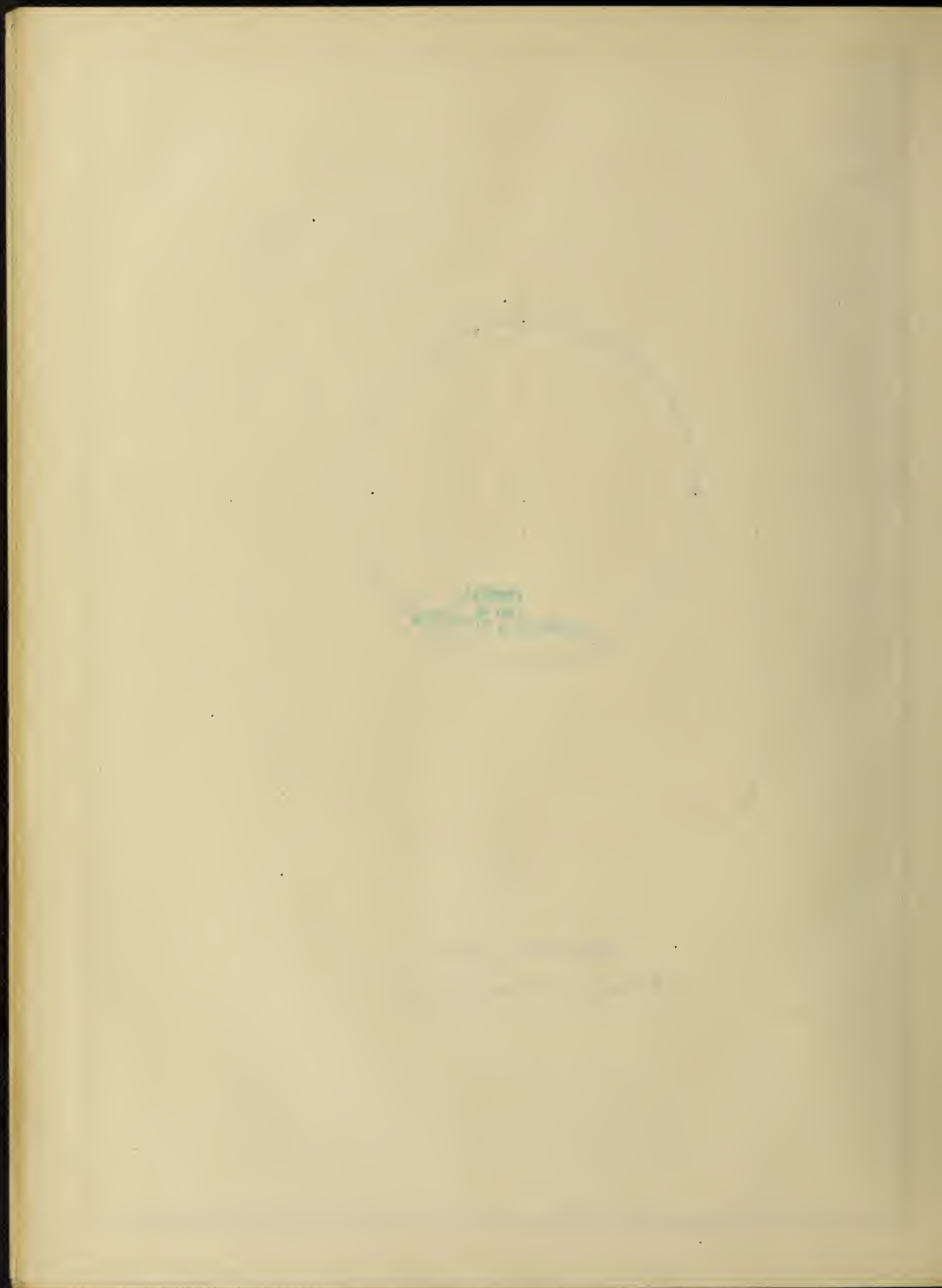
Degree	Average	Difference	Factor	Candle	C.P.per	Corrected
	cm.	cm.		Power	cu. ft.	
0	238	740	9.7	155	18.9	25.2
30	232	746	10.4	165	20.2	26.9
60	231	747	10.5	167	20.4	27.2
90	229	749	10.7	171	20.8	27.7
120	227	751	10.9	175	21.3	28.4
150	226	752	11.0	177	21.5	29.0
180	225	753	11.2	179	21.8	29.2
210	227	751	10.9	175	21.3	28.4
240	227	751	10.9	175	21.3	28.4
270	229	749	10.7	171	20.8	27.7
300	229	749	10.7	171	20.8	27.7
330	230	748	10.6	169	20.6	<u>27.5</u>
Average -						28.9



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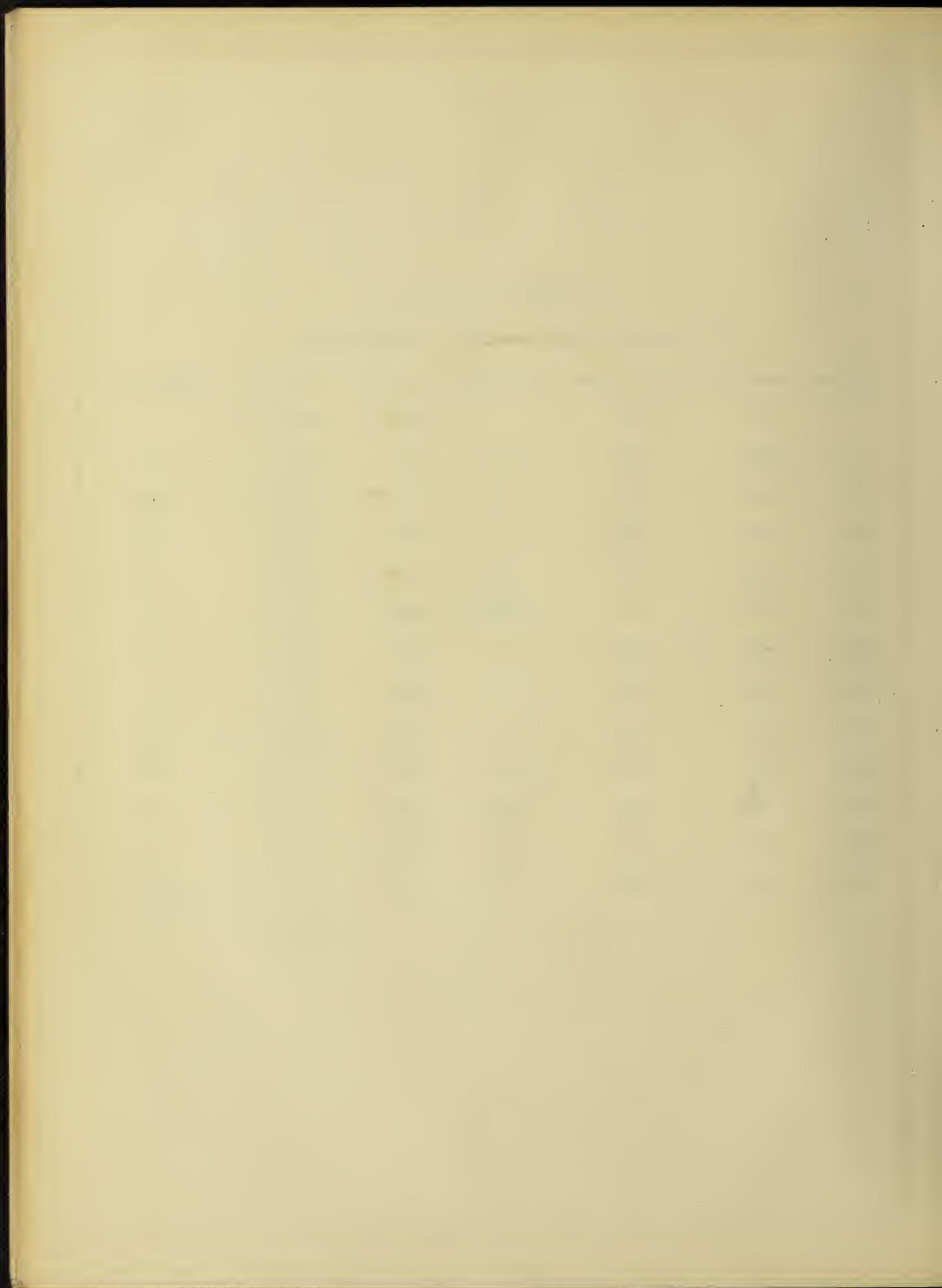




Horizontal Circles

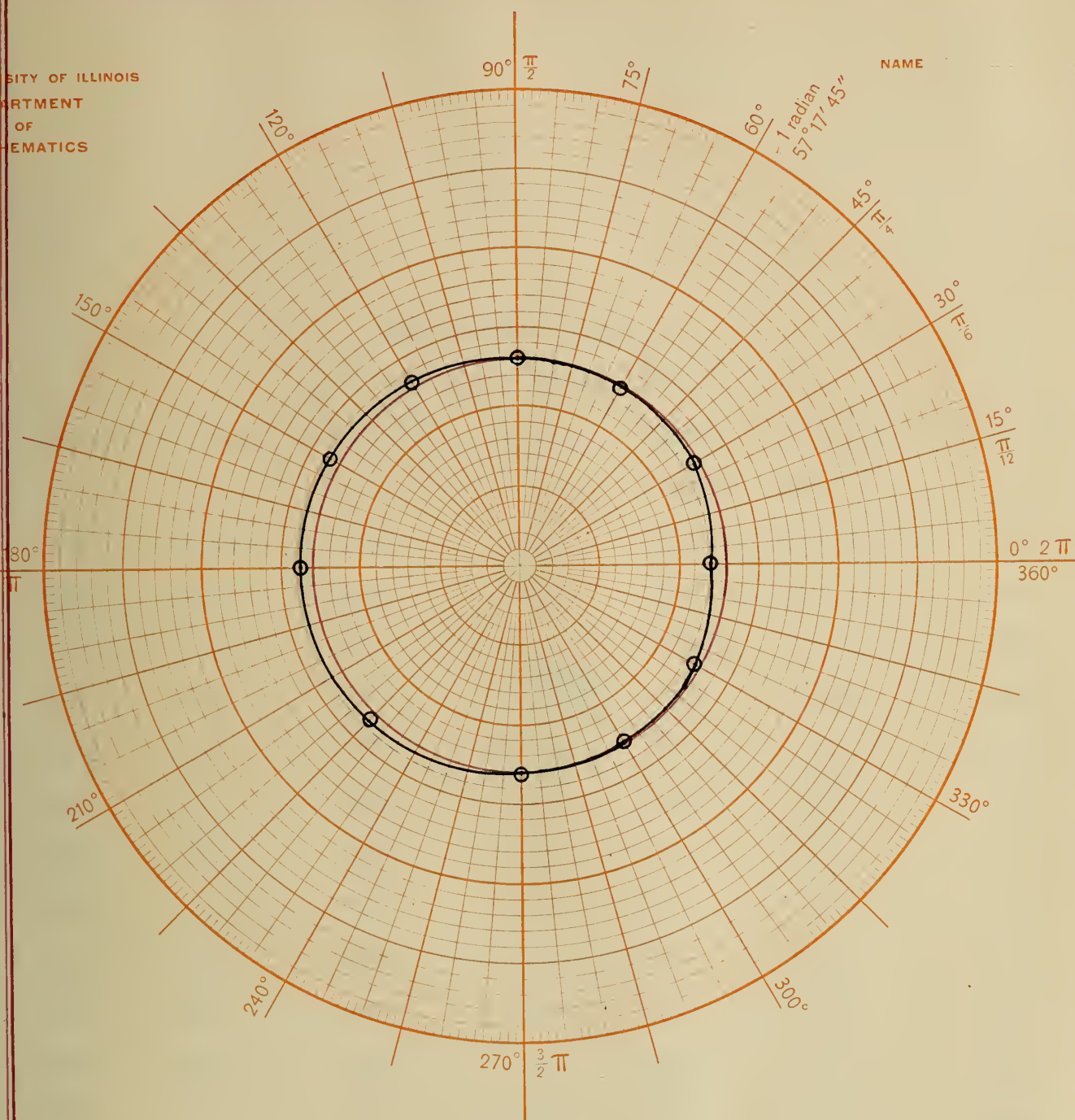
90 Degrees from Vertical - Horizontal

Degree	Average	Difference	Factor	Candle	C.P.per	Corrected
	cm.	cm.		Power	cu. ft.	
0	242	736	9.3	148	18.1	24.0
30	238	740	9.7	155	18.9	25.2
60	237	741	9.8	157	19.2	25.6
90	235	743	9.9	160	19.6	26.0
120	233	745	10.3	163	20.0	26.6
150	231	747	10.5	167	20.4	27.2
180	230	748	10.6	169	20.6	27.5
210	232	746	10.4	165	20.2	26.9
240	234	744	10.2	162	19.8	26.3
270	236	742	9.95	159	19.4	25.7
300	238	740	9.7	155	18.9	25.2
330	238	740	9.7	155	18.9	<u>25.2</u>
Average -						26.0



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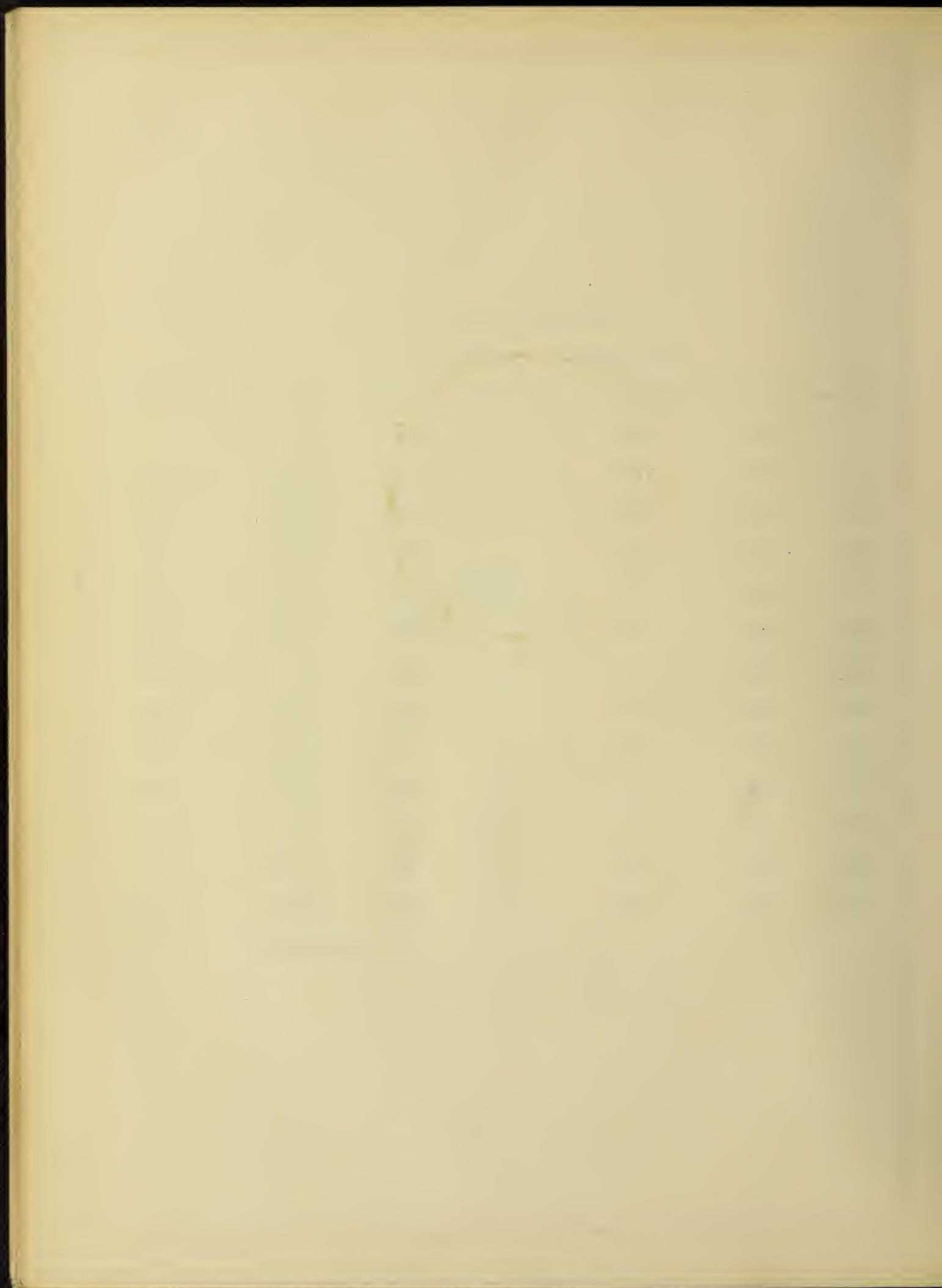


Horizontal Circle
90 Degrees from Vertical



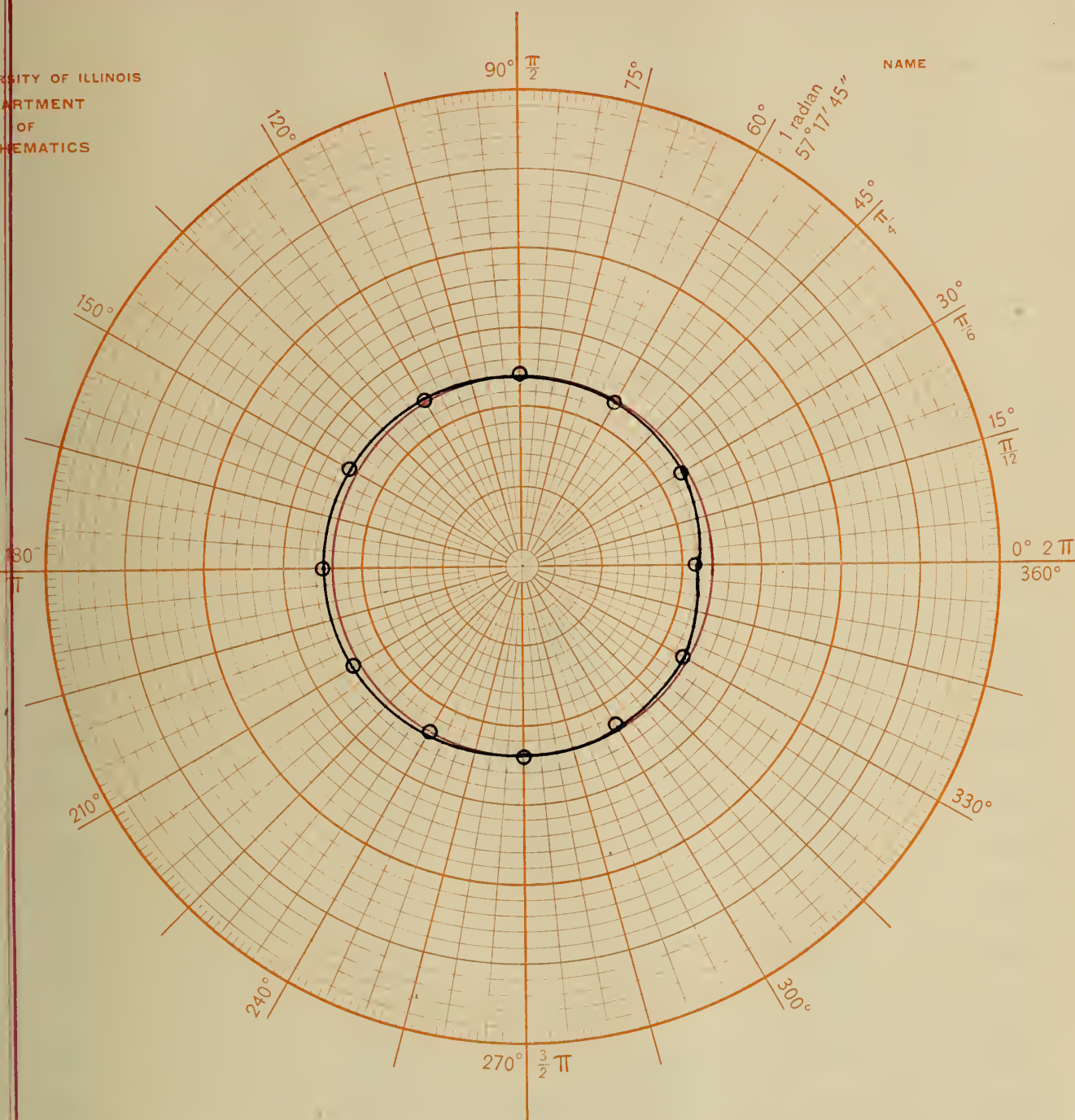
Horizontal Circles
100 Degrees from Vertical

Degree	Average cm.	Difference cm.	Factor	Candle Power	C.P. per cu. ft.	Corrected
0	251	727	8.4	134	16.4	21.8
30	245	733	8.9	143	17.4	23.1
60	243	735	9.1	146	17.8	23.7
90	241	737	9.35	150	18.3	24.3
120	241	737	9.35	150	18.3	24.3
150	239	739	9.6	153	18.7	24.9
180	238	740	9.7	155	18.9	25.2
210	240	738	9.5	152	18.5	24.7
240	242	736	9.25	148	18.1	24.0
270	242	736	9.25	148	18.1	24.0
300	245	733	8.9	143	17.4	23.1
330	244	734	9.0	144	17.6	<u>23.4</u>
Average -						23.7



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Horizontal Circle
100 Degrees from Vertical

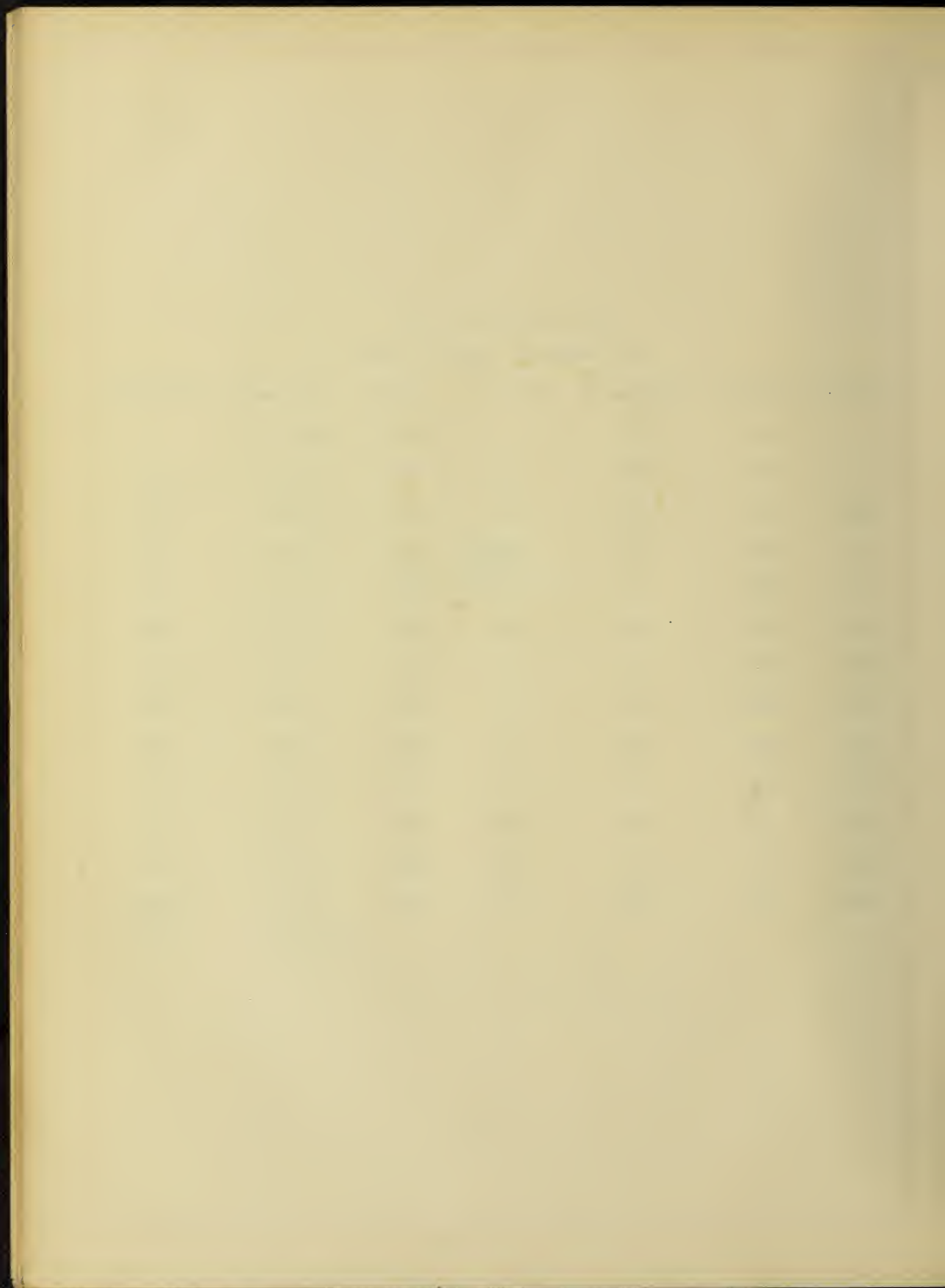


THE UNIVERSITY OF CHICAGO
LIBRARY

Horizontal Circles

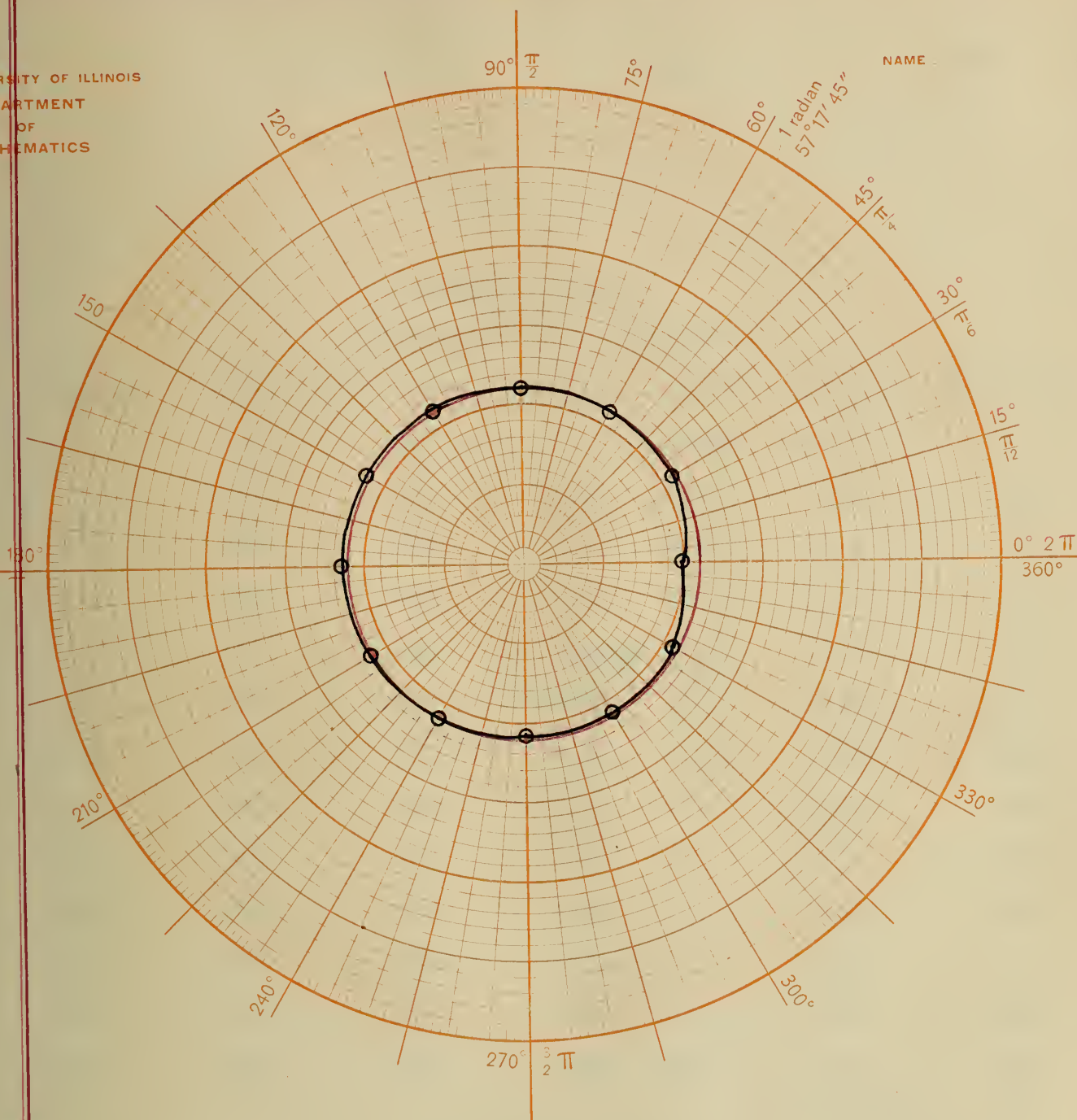
110 Degrees from Vertical

Degree	Average	Difference	Factor	Candle	C.P.per	Corrected
	cm.	cm.		Power	cu. ft.	
0	260	718	7.65	122	14.9	19.8
30	253	725	8.2	132	16.1	21.4
60	251	727	8.4	134	16.4	21.8
90	250	728	8.45	135	16.5	22.0
120	249	729	8.5	137	16.7	22.2
150	247	731	8.7	140	17.1	22.6
180	246	732	8.8	142	17.3	22.9
210	248	730	8.6	138	16.9	22.4
240	249	729	8.5	137	16.7	22.2
270	250	728	8.45	135	16.5	22.0
300	251	727	8.4	134	16.4	21.8
330	253	725	8.2	132	16.1	<u>21.4</u>
Average -						22.0



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DEPARTMENT
OF
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NAME _____



Horizontal Circle

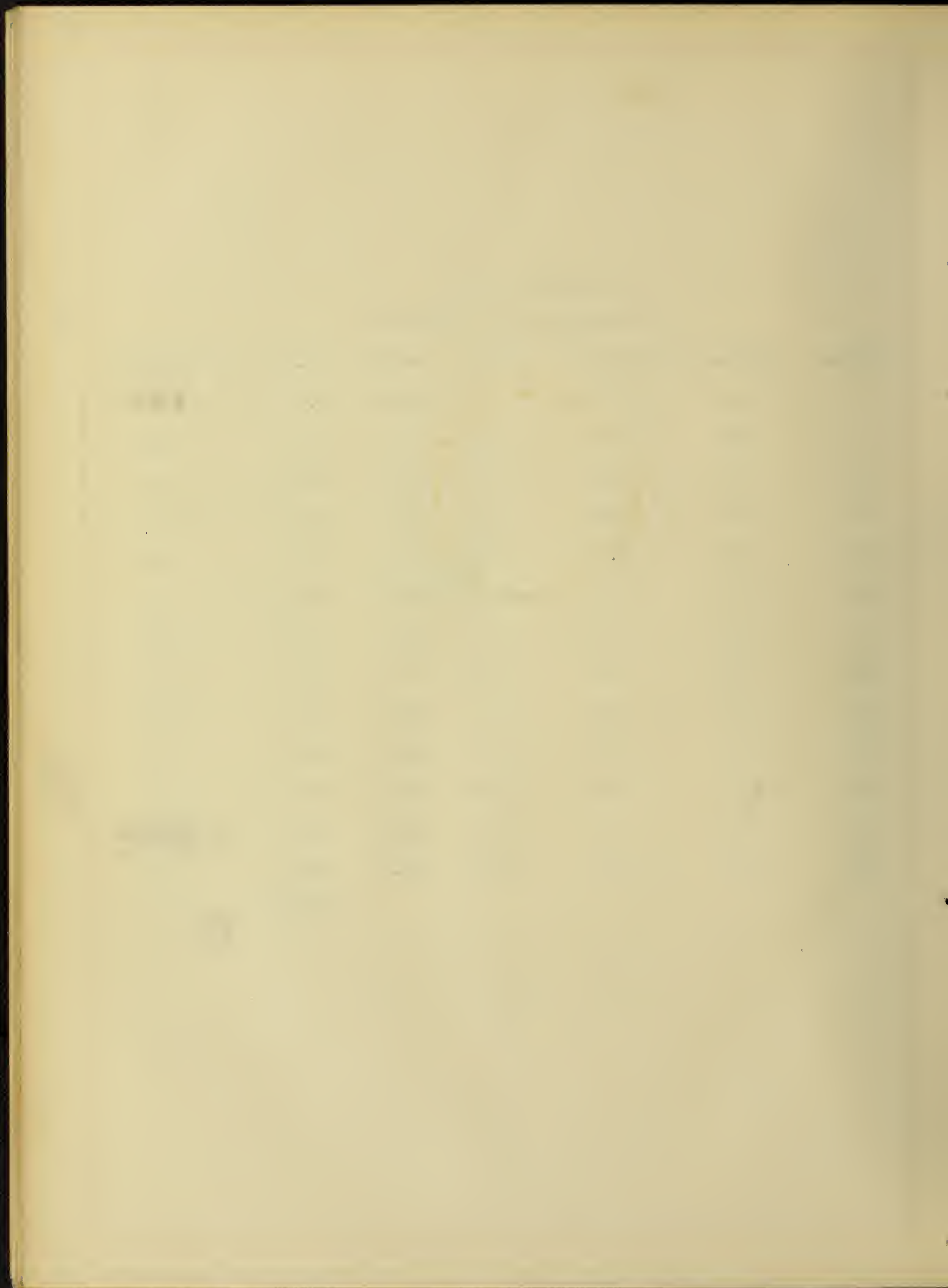
110 Degrees from Vertical



Horizontal Circle

120 Degrees from Vertical

Degree	Average	Difference	Factor	Candle	C.P.per	Corrected
	cm.	cm.		Power	cu. ft.	
0	286	692	5.82	93.5	11.4	16.7
30	276	702	6.45	103.5	12.4	17.0
60	275	703	6.55	105.0	12.8	17.2
90	274	704	6.60	106.0	12.9	17.8
120	271	707	6.85	110.0	13.4	18.1
150	269	709	6.96	113.0	13.6	18.0
180	268	710	6.93	111.0	13.5	17.9
210	270	708	6.90	110.7	13.5	17.4
240	272	706	6.80	108.0	13.2	17.0
270	275	703	6.55	105.0	12.8	17.0
300	275	703	6.55	105.0	12.8	16.5
330	278	700	6.35	102.0	12.4	15.2
Average -						17.1





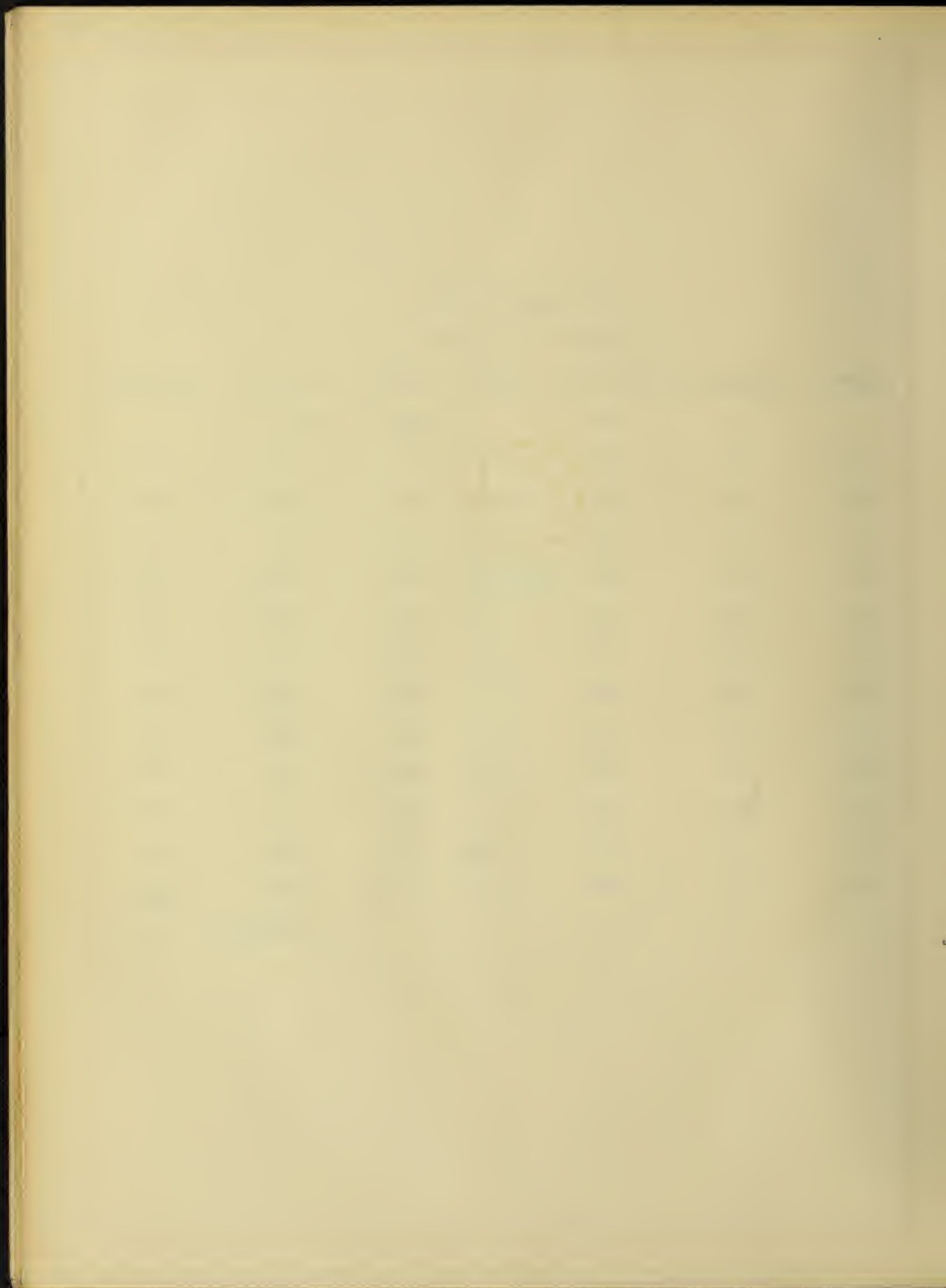
Horizontal Circle
120 Degrees from Vertical

LIBRARY
OF THE
MUSEUM OF NATURAL HISTORY

Horizontal Circle

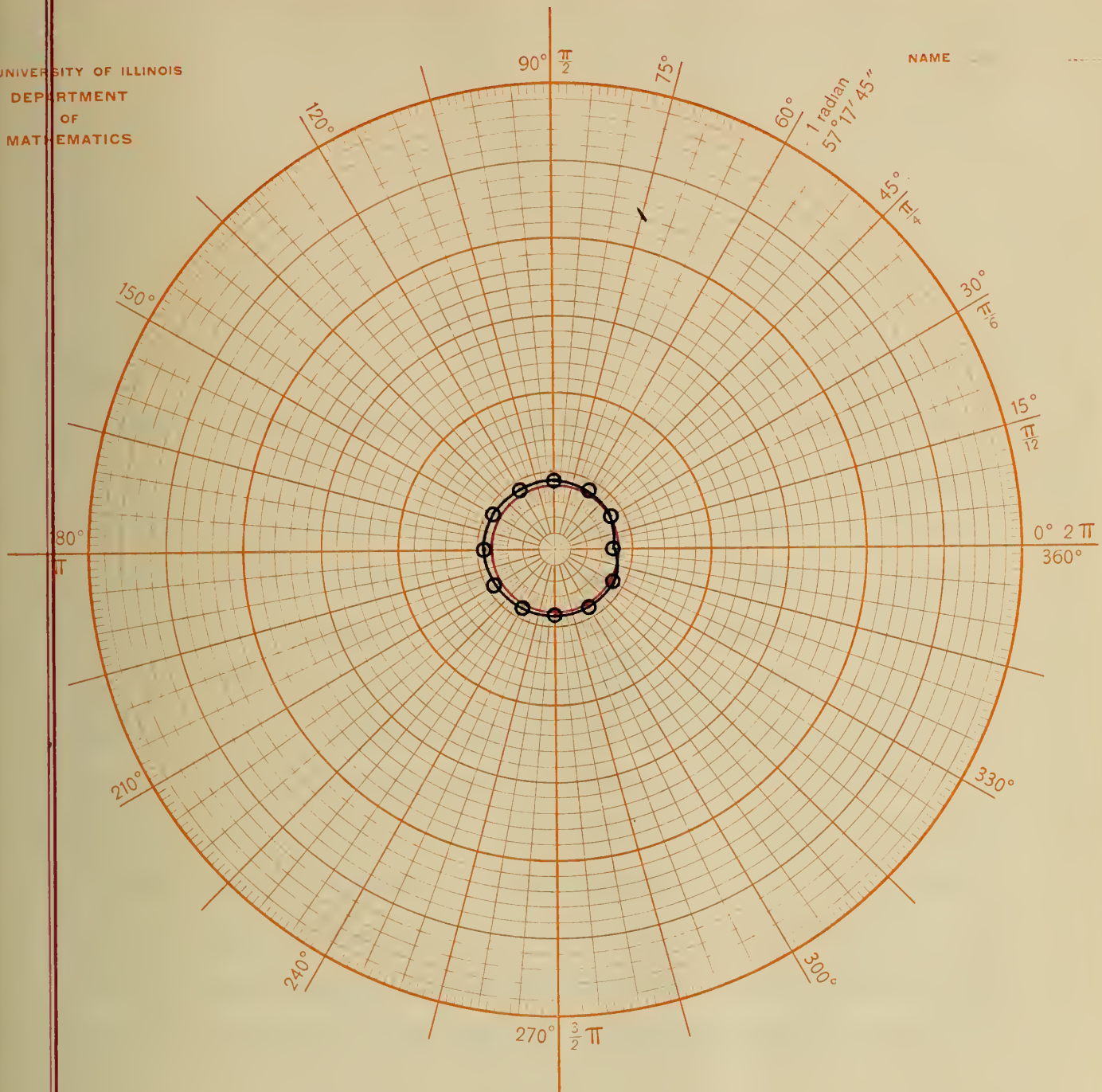
130 Degrees from Vertical

Degree	Average cm.	Difference cm.	Factor	Candle Power	C.P. per cu.ft.	Corrected
0	368	612	2.82	45.0	5.5	7.3
30	352	626	3.15	51.0	6.15	8.2
60	350	628	3.25	52.0	6.3	8.4
90	347	631	3.30	52.8	6.45	8.6
120	345	633	3.26	53.8	6.58	7.7
150	343	635	3.43	54.8	6.7	8.9
180	340	638	3.52	56.2	6.85	9.2
210	341	637	3.49	55.7	6.79	9.0
240	346	632	3.34	53.2	6.5	8.65
270	348	630	3.27	52.5	6.4	8.57
300	349	629	3.26	52.4	6.4	8.5
330	351	627	3.20	51.0	6.22	<u>8.3</u>
Average -						8.1



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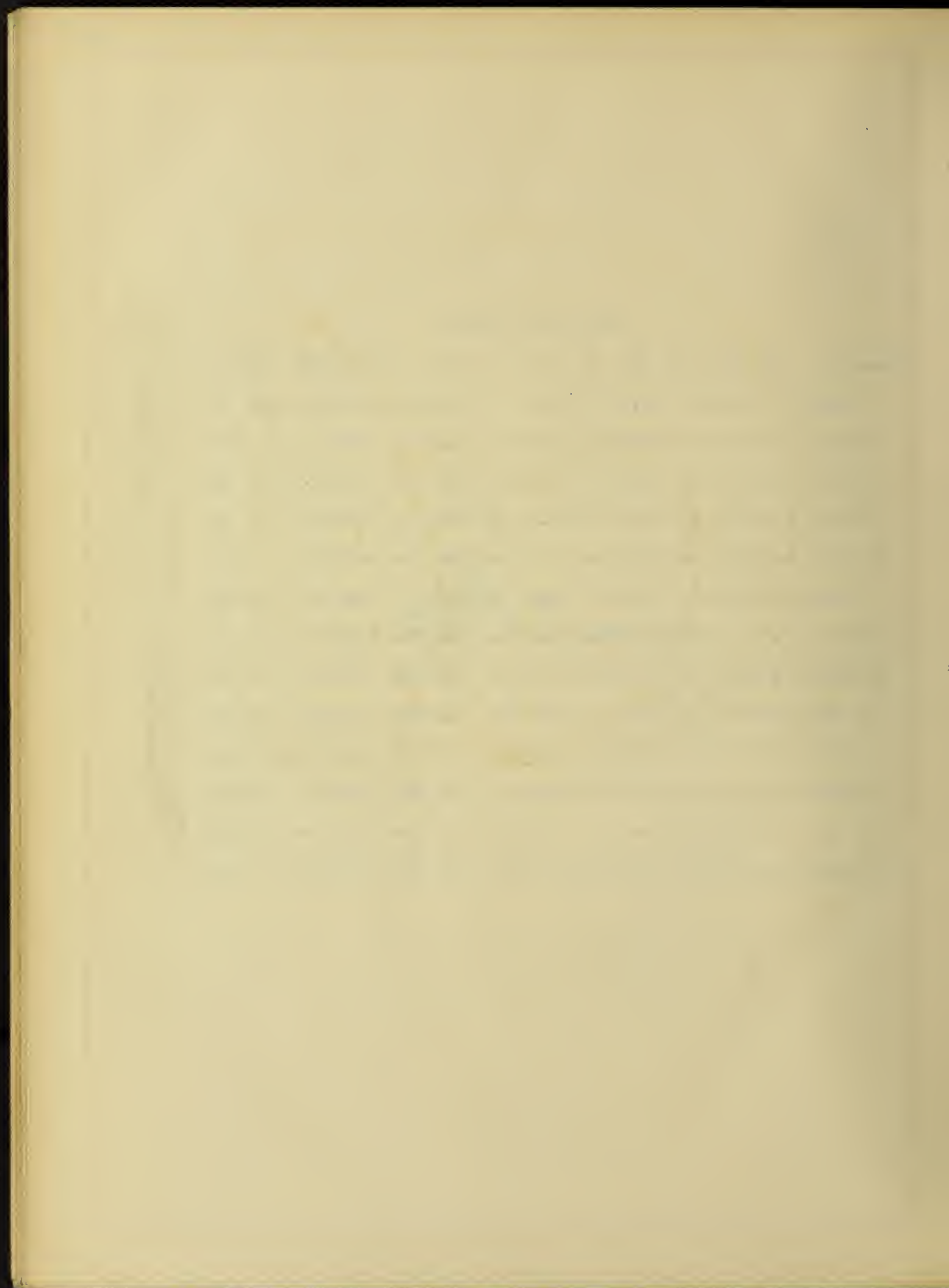
Horizontal Circle

130 Degrees from Vertical



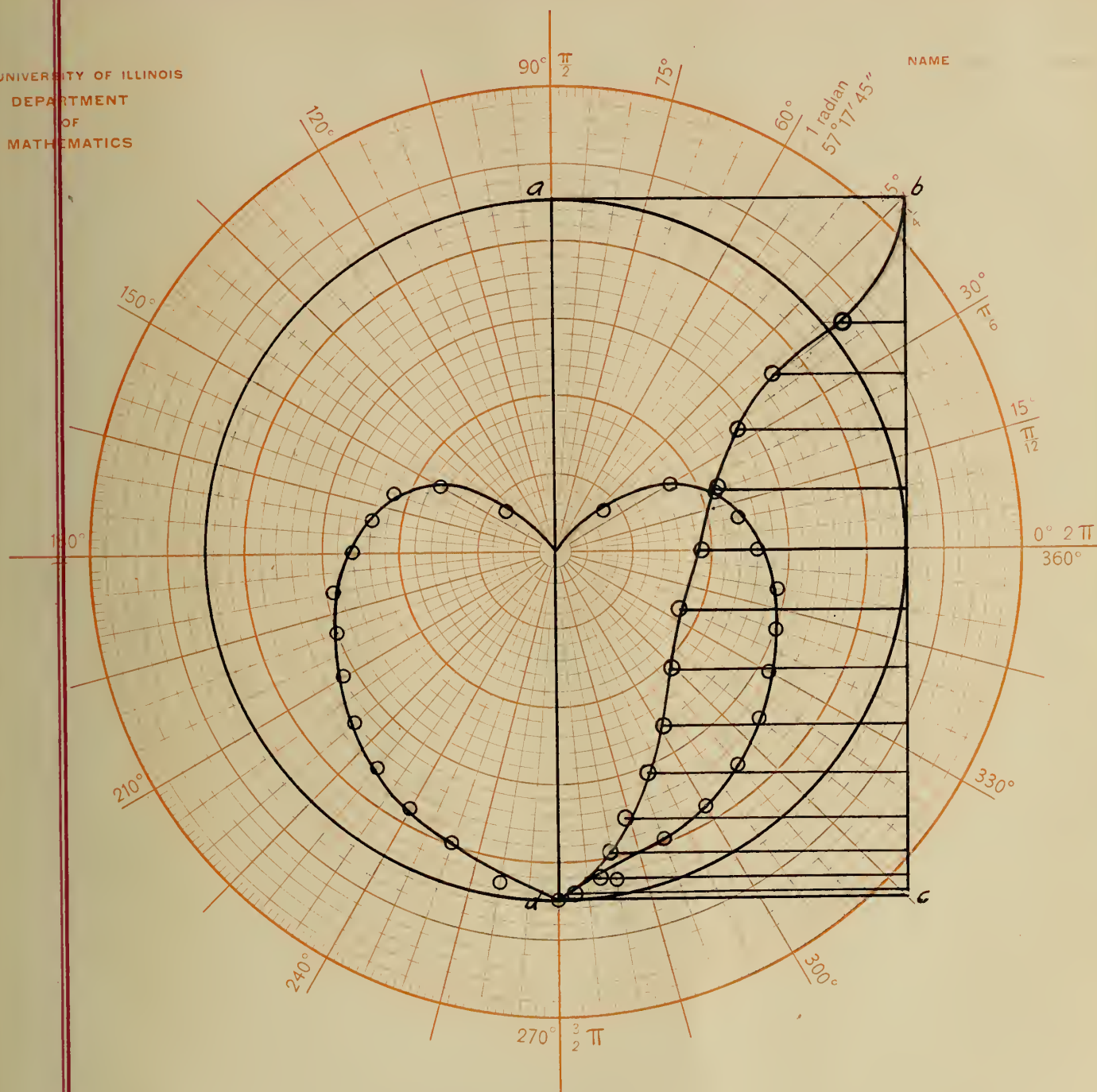
Vertical Circles

Deg.	0	10	20	30	40	50	60	70	80	90	100	110	120	130
0	404	390	370	343	323	320	295	277	252	240	218	198	152	730
30	430	418	393	370	348	323	308	295	269	252	251	214	167	820
60	447	423	393	376	356	321	312	295	272	256	237	218	170	840
90	447	423	395	376	362	348	317	298	277	260	243	220	172	861
120	458	430	404	453	370	356	317	302	284	266	243	222	175	872
150	464	436	404	393	375	369	320	308	290	272	249	226	181	890
180	472	443	418	404	370	348	323	312	292	275	252	229	180	920
210	468	436	418	395	370	343	323	308	284	269	247	224	179	900
240	458	430	404	390	362	321	320	305	284	263	240	222	174	865
270	440	423	395	383	369	321	320	295	277	257	240	220	170	857
300	436	418	395	376	356	327	312	298	277	252	231	218	170	853
330	430	404	393	370	348	327	312	295	275	252	234	214	165	830
Ave	448	429	393	381	360	338	314	300	289	260	239	220	171	810



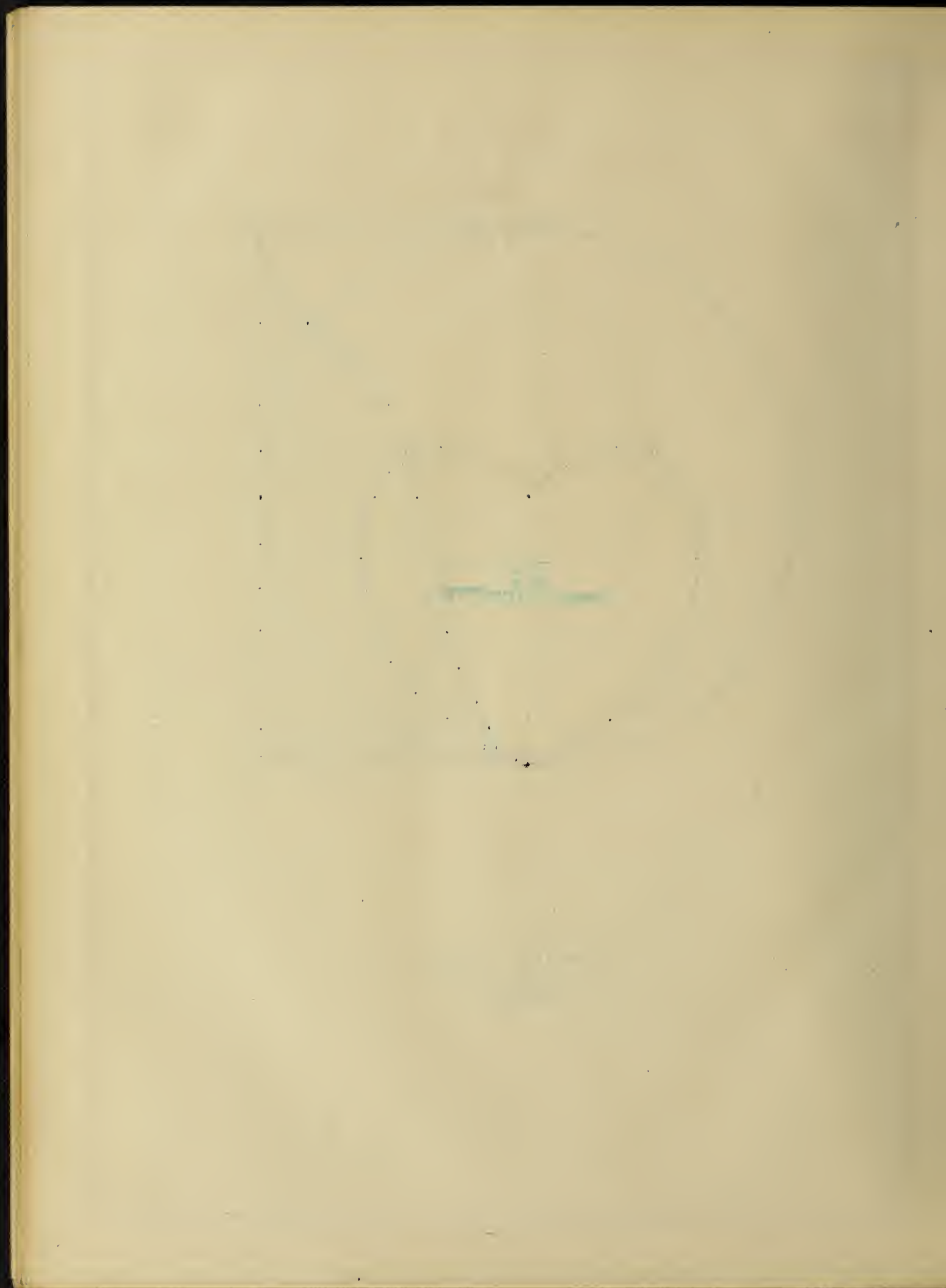
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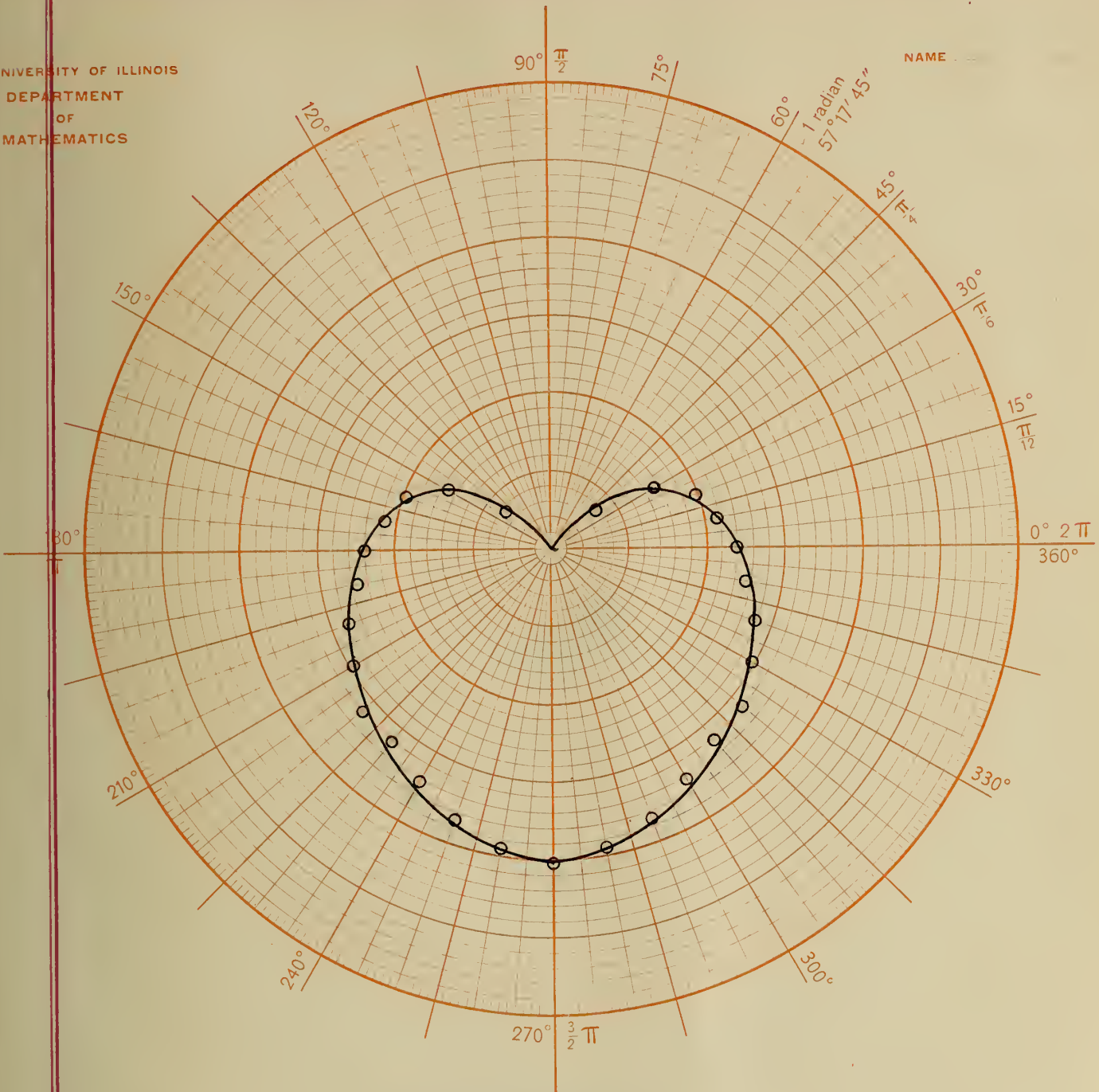
Vertical Circle

Average



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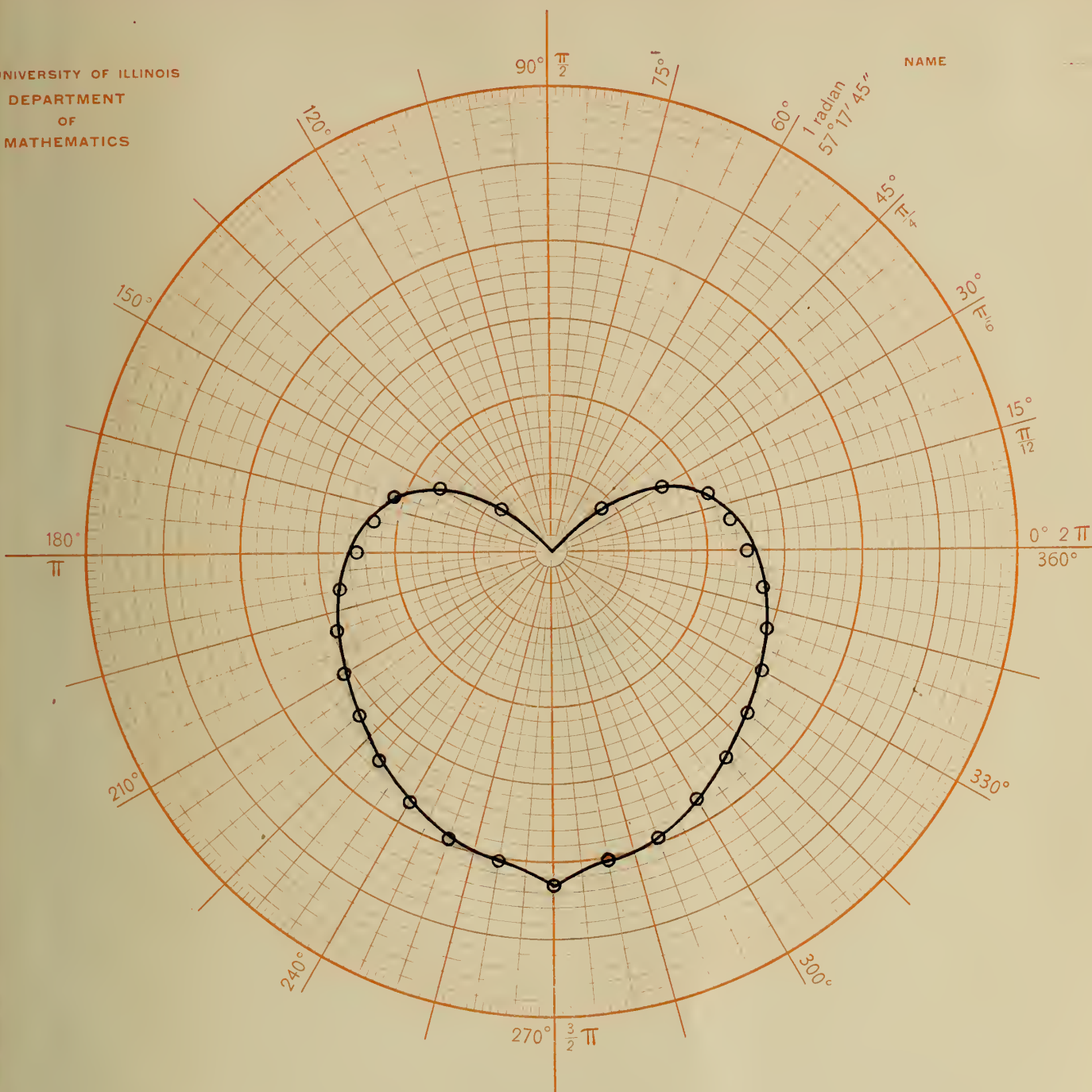
Vertical Circles

0 Degrees

1887
1888
1889

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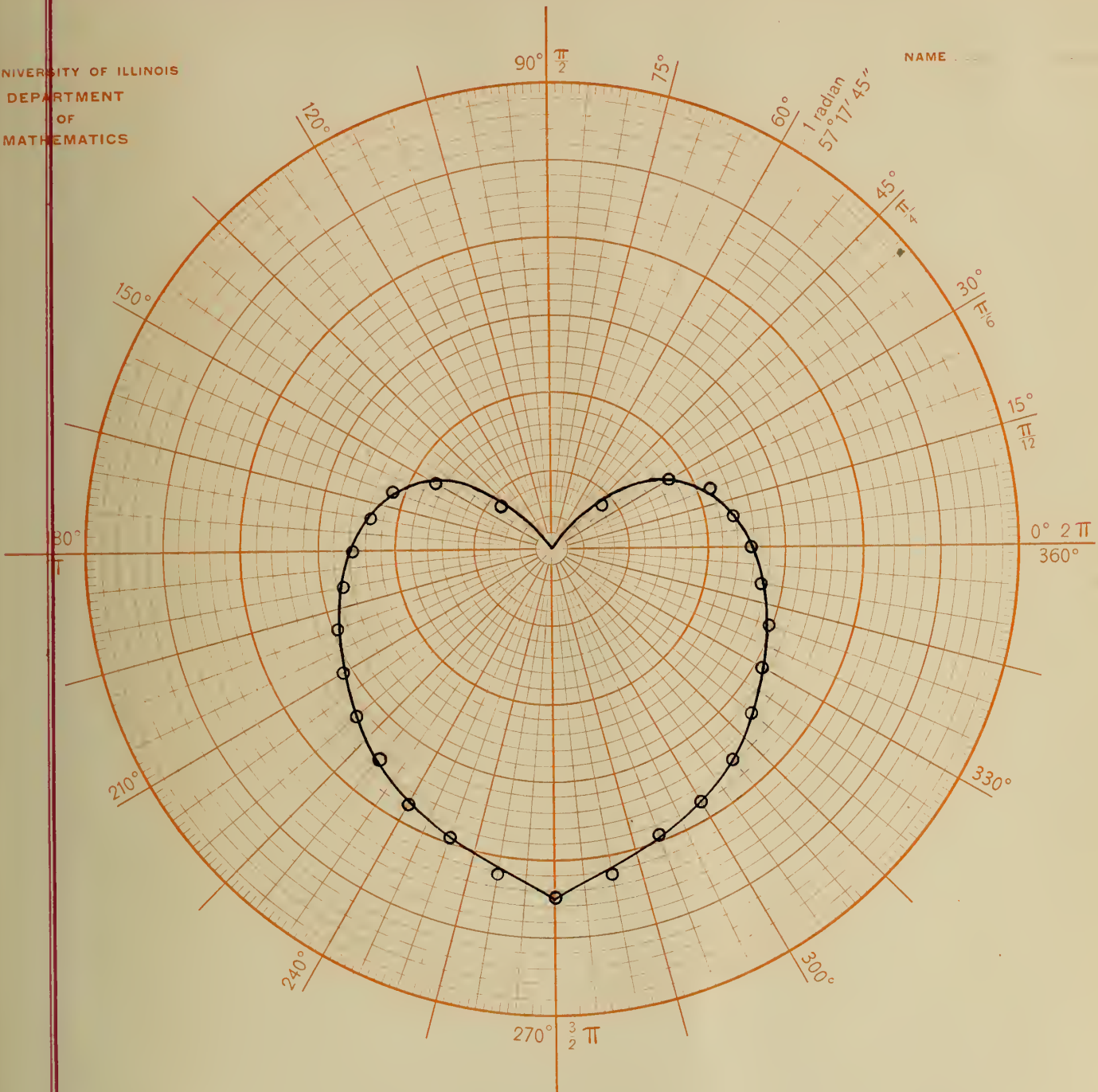
Vertical Circle

30 Degrees



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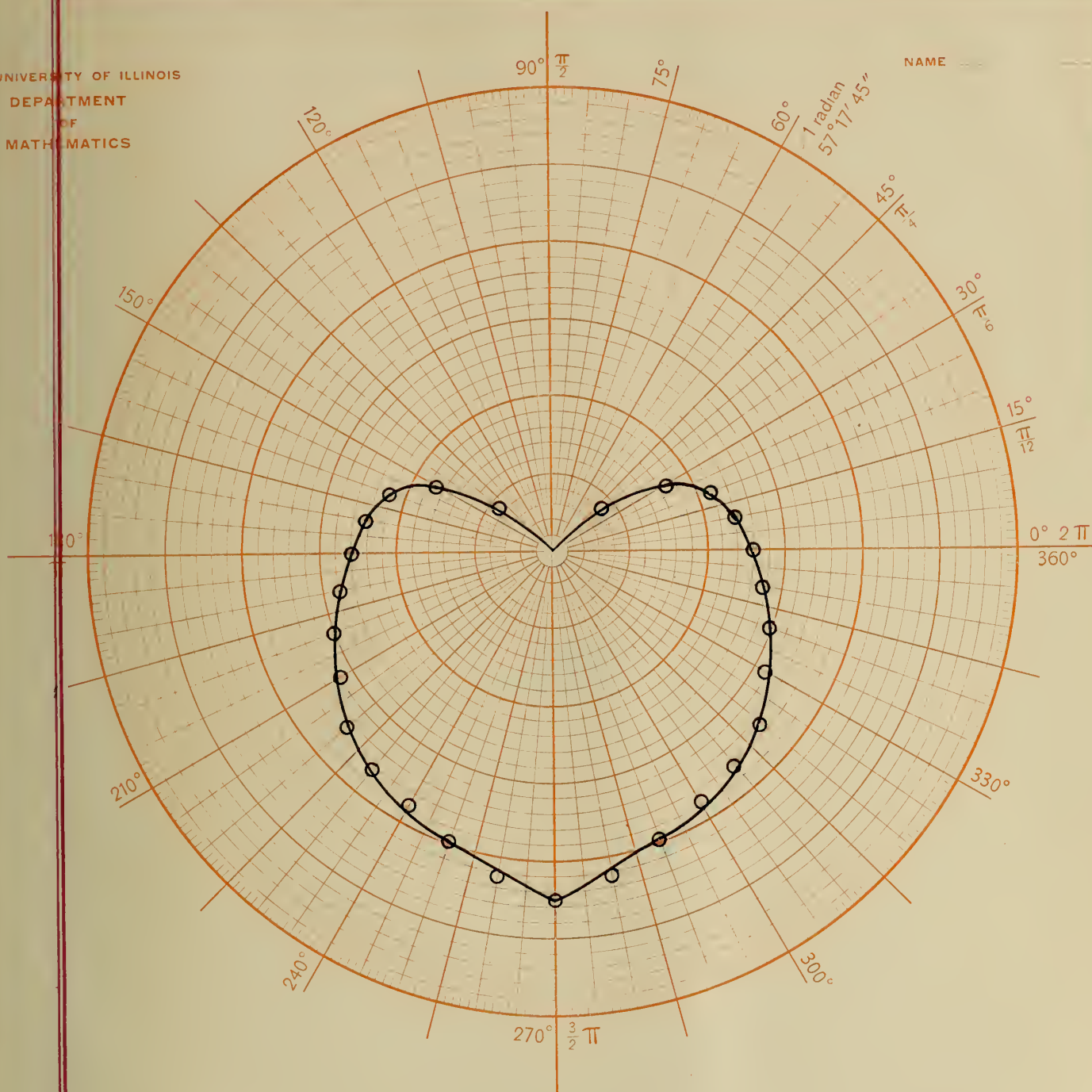
Vertical Circle

60 Degrees

1
2

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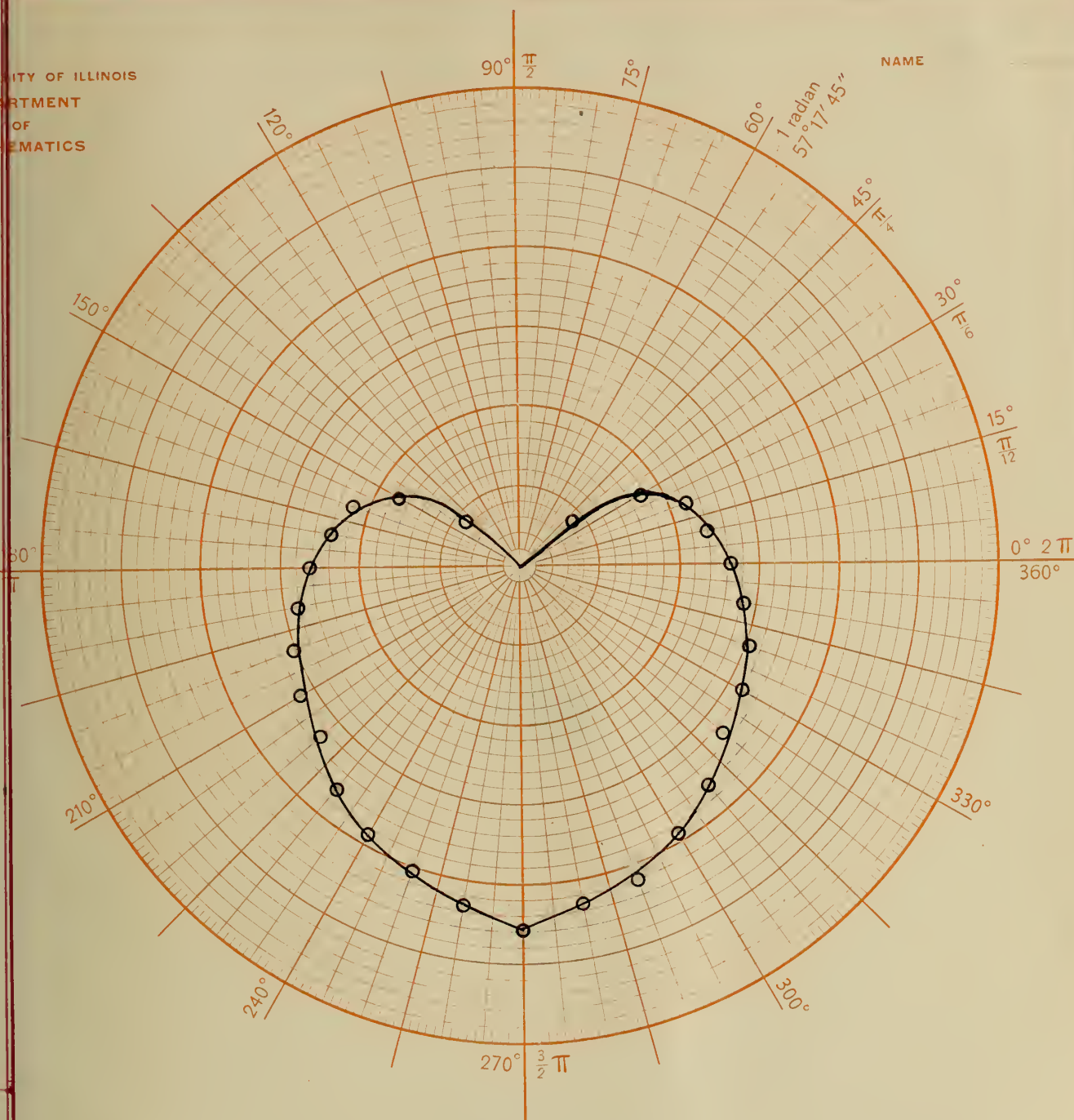
Vertical Circle

90 Degrees



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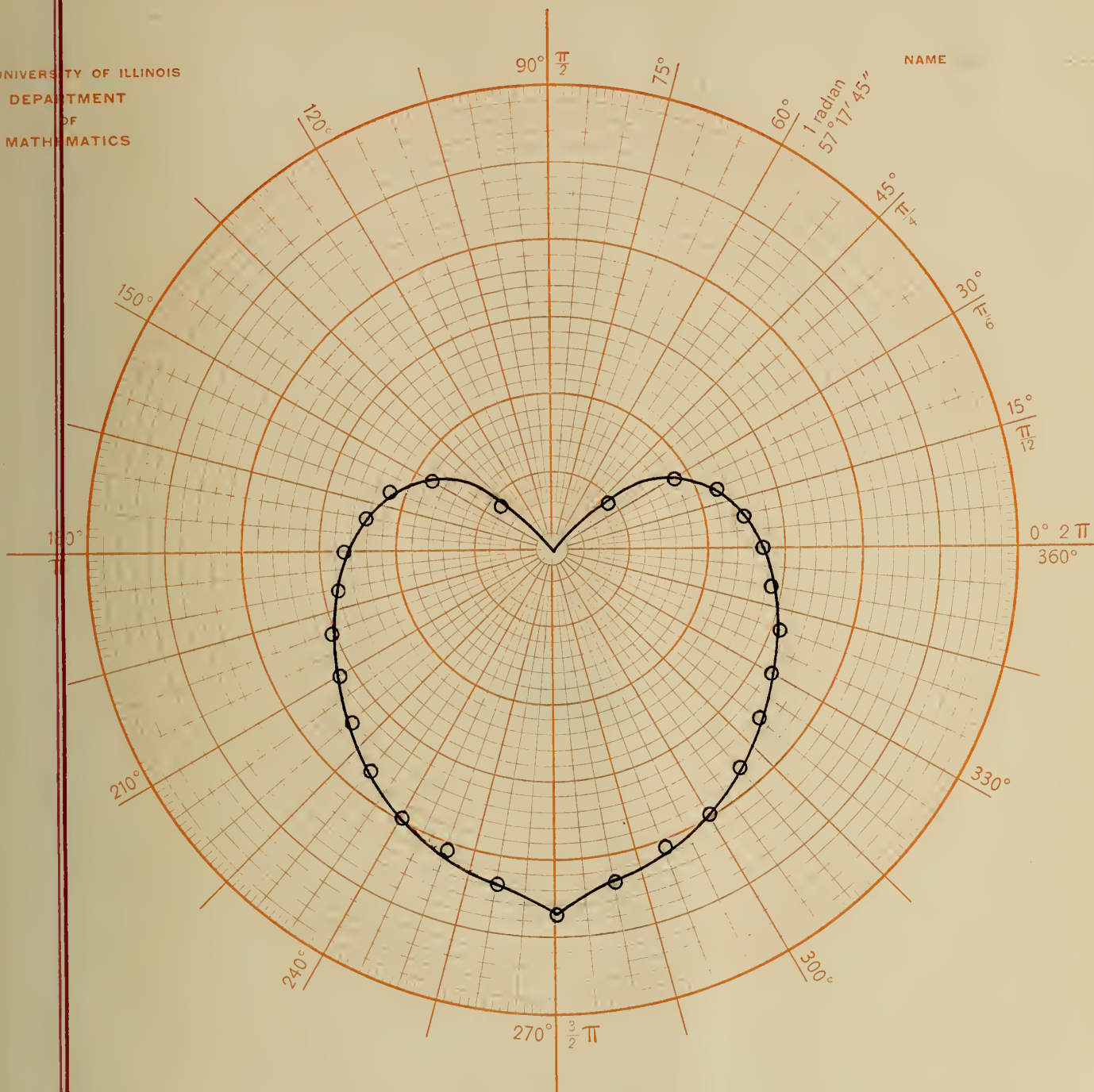
Vertical Circle

120 Degrees



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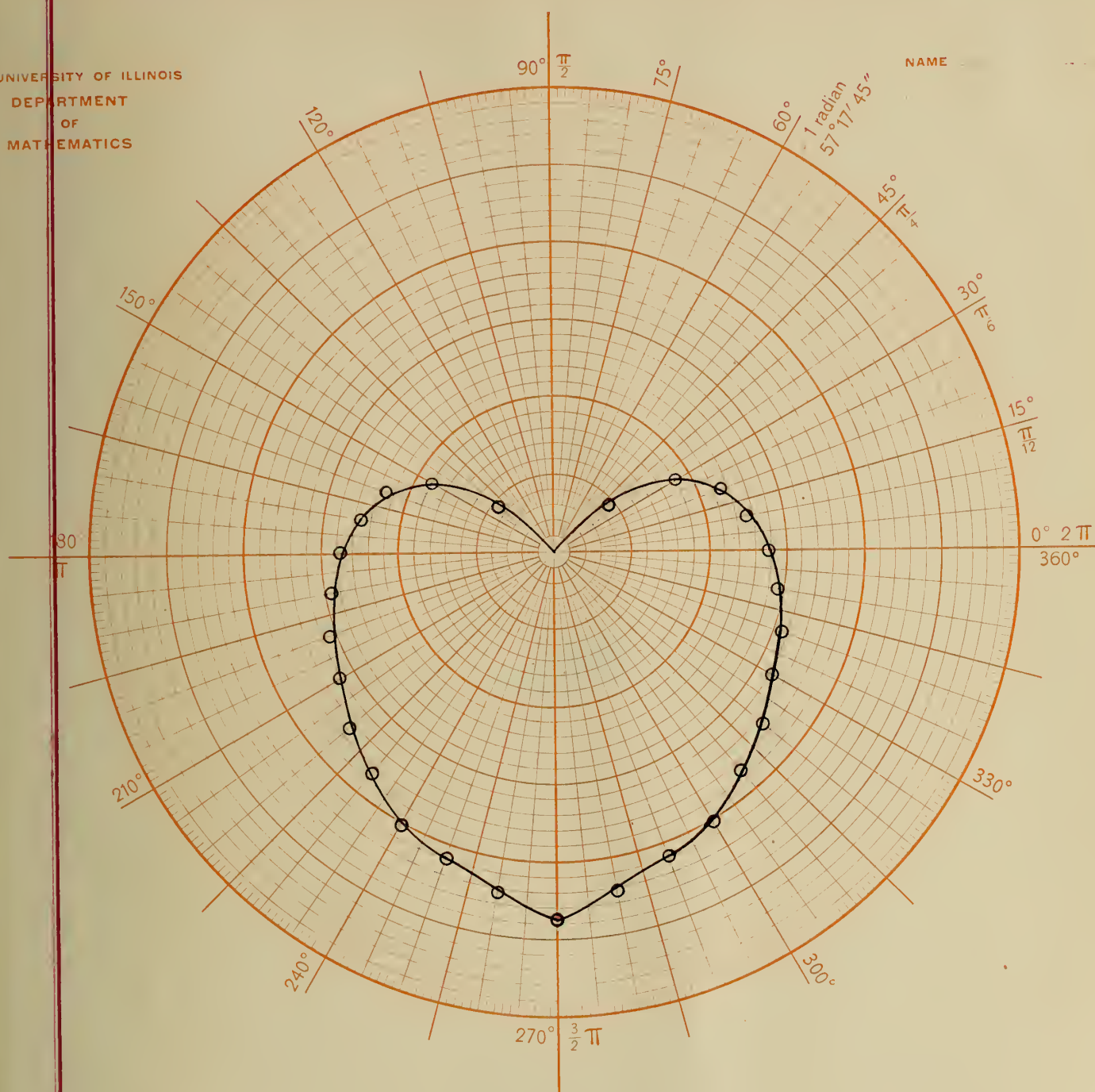
Vertical Circle

150 Degrees



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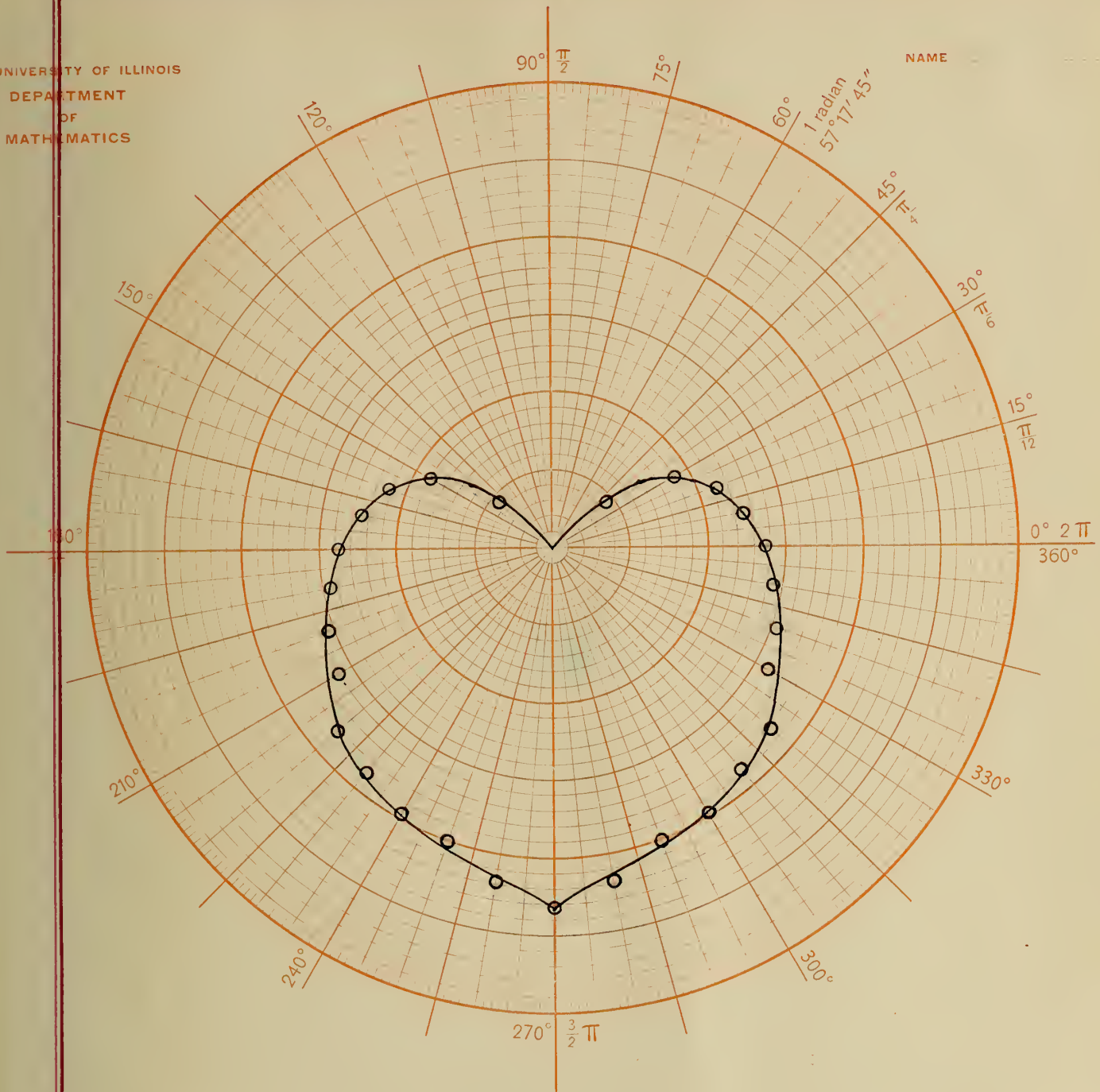
Vertical Circle

180 Degrees



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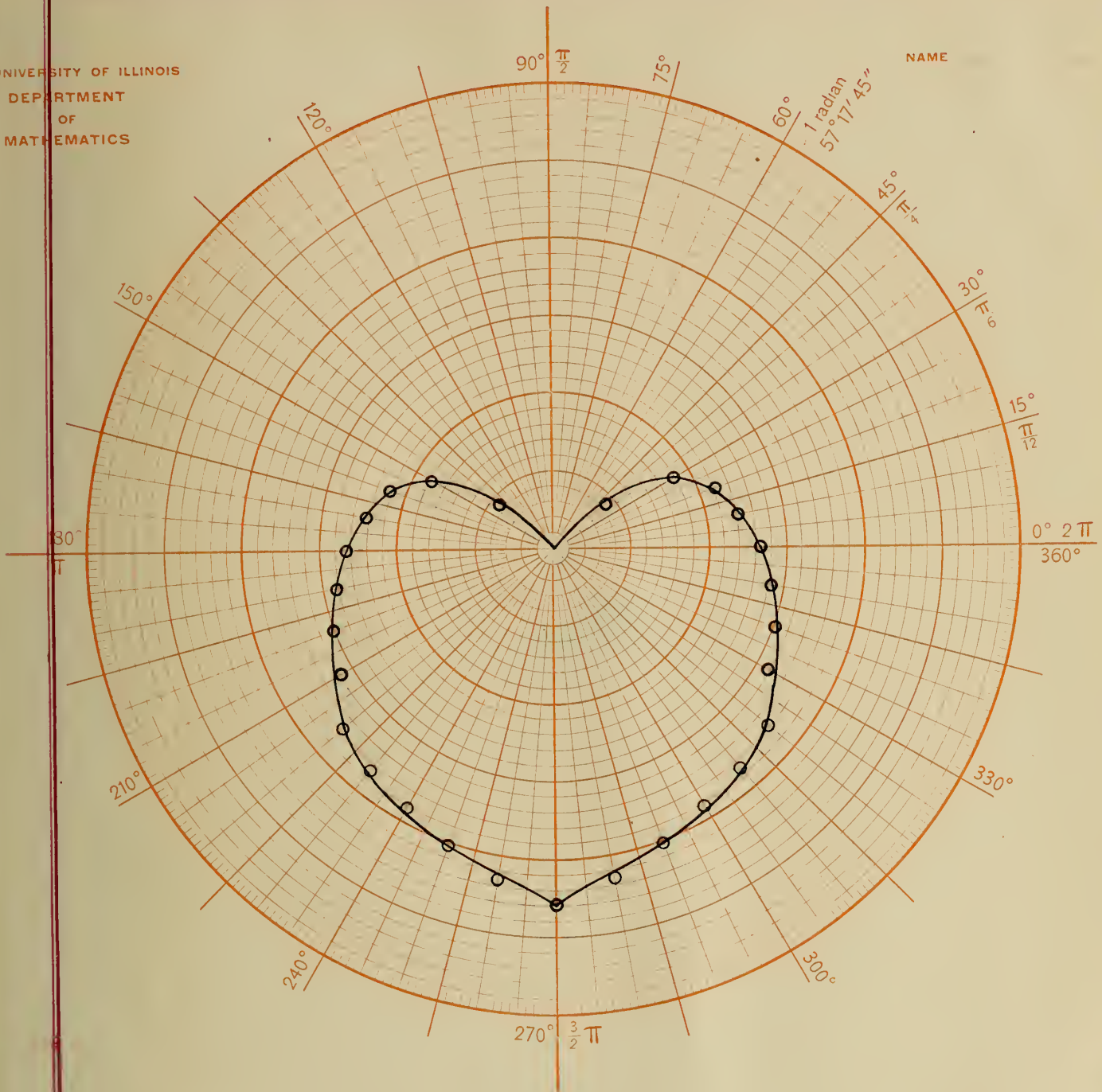
Vertical Circle

210 Degrees



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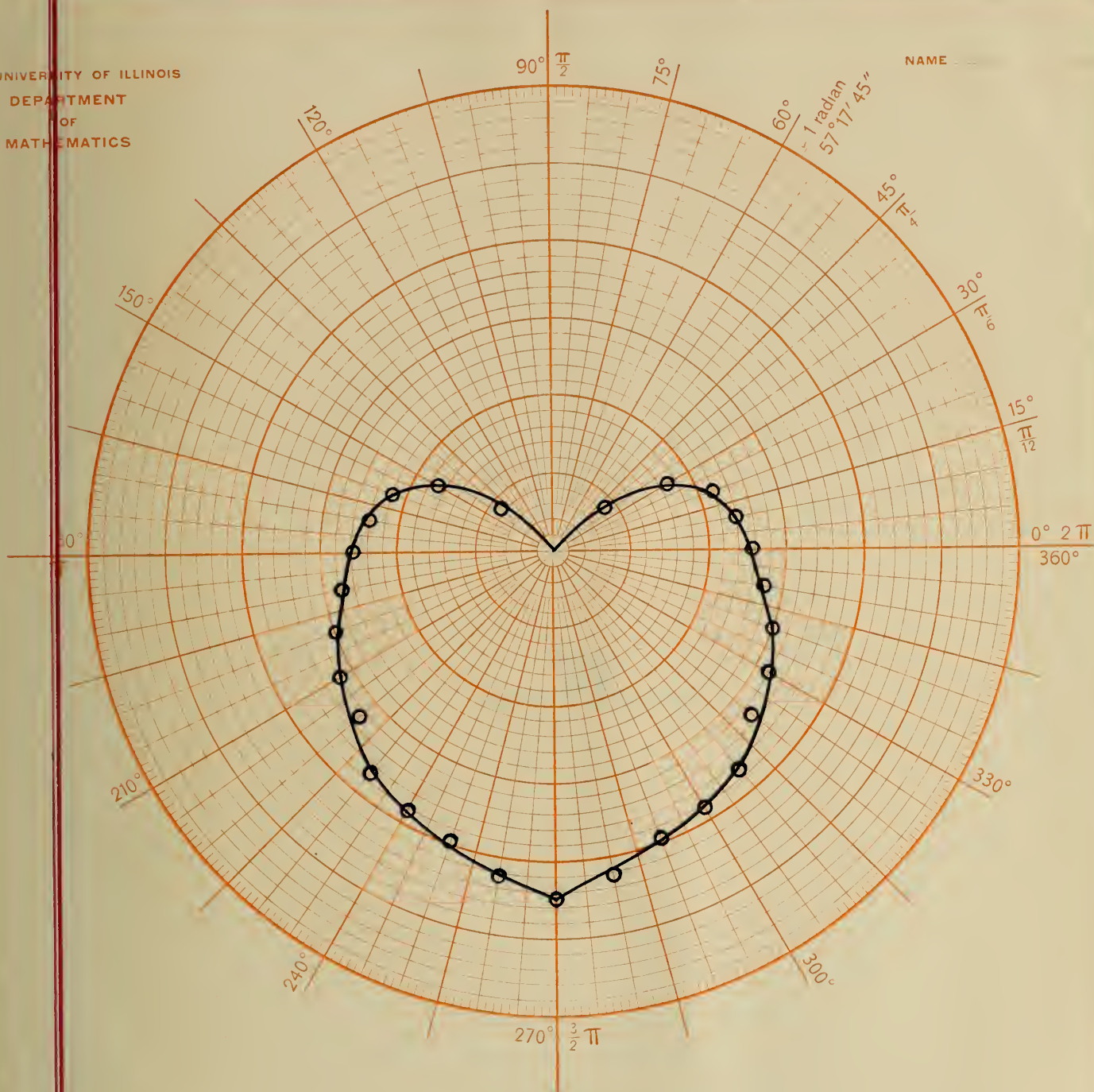
Vertical Circle

240 Degrees



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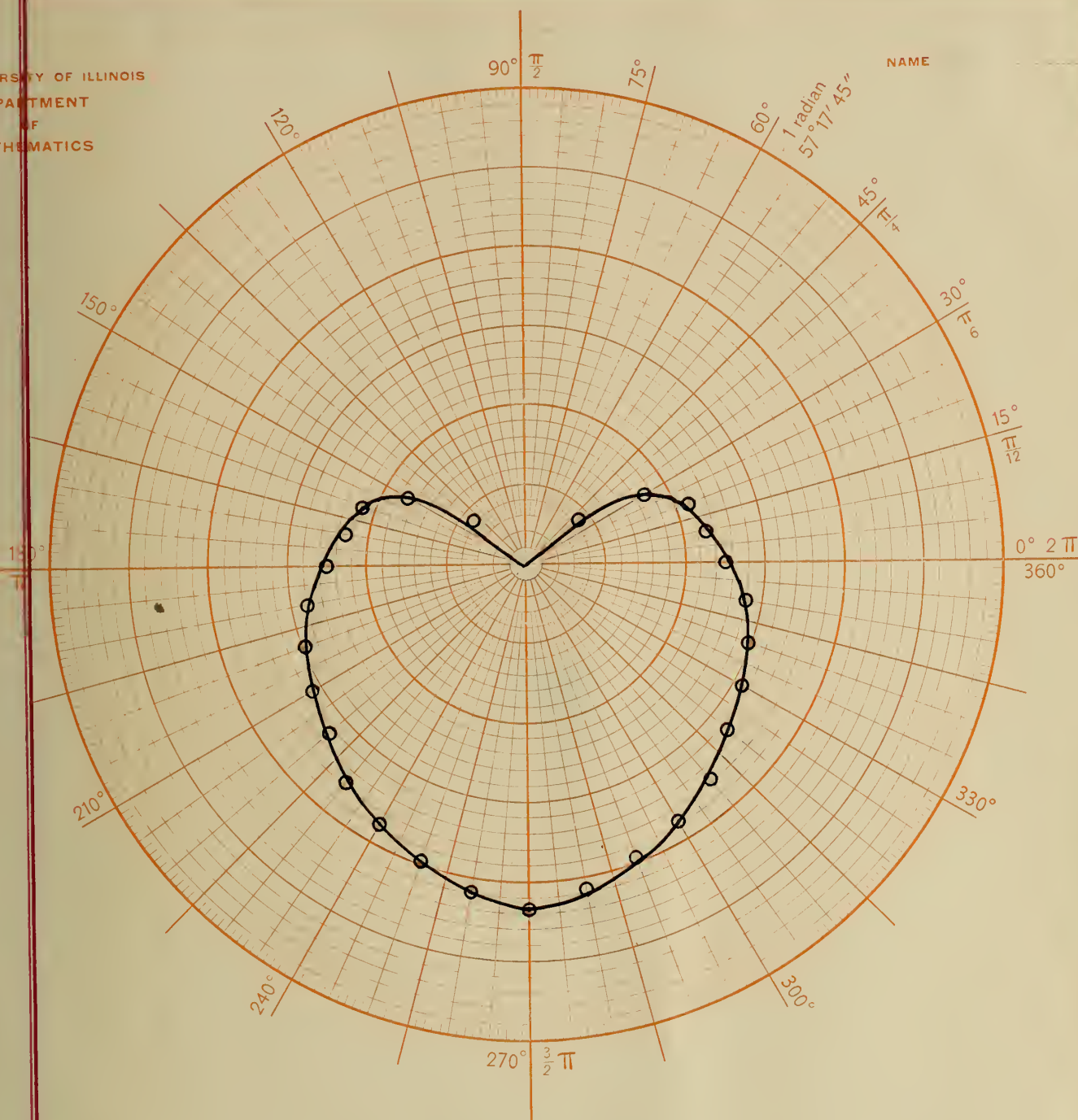
Vertical Circle

270 Degrees



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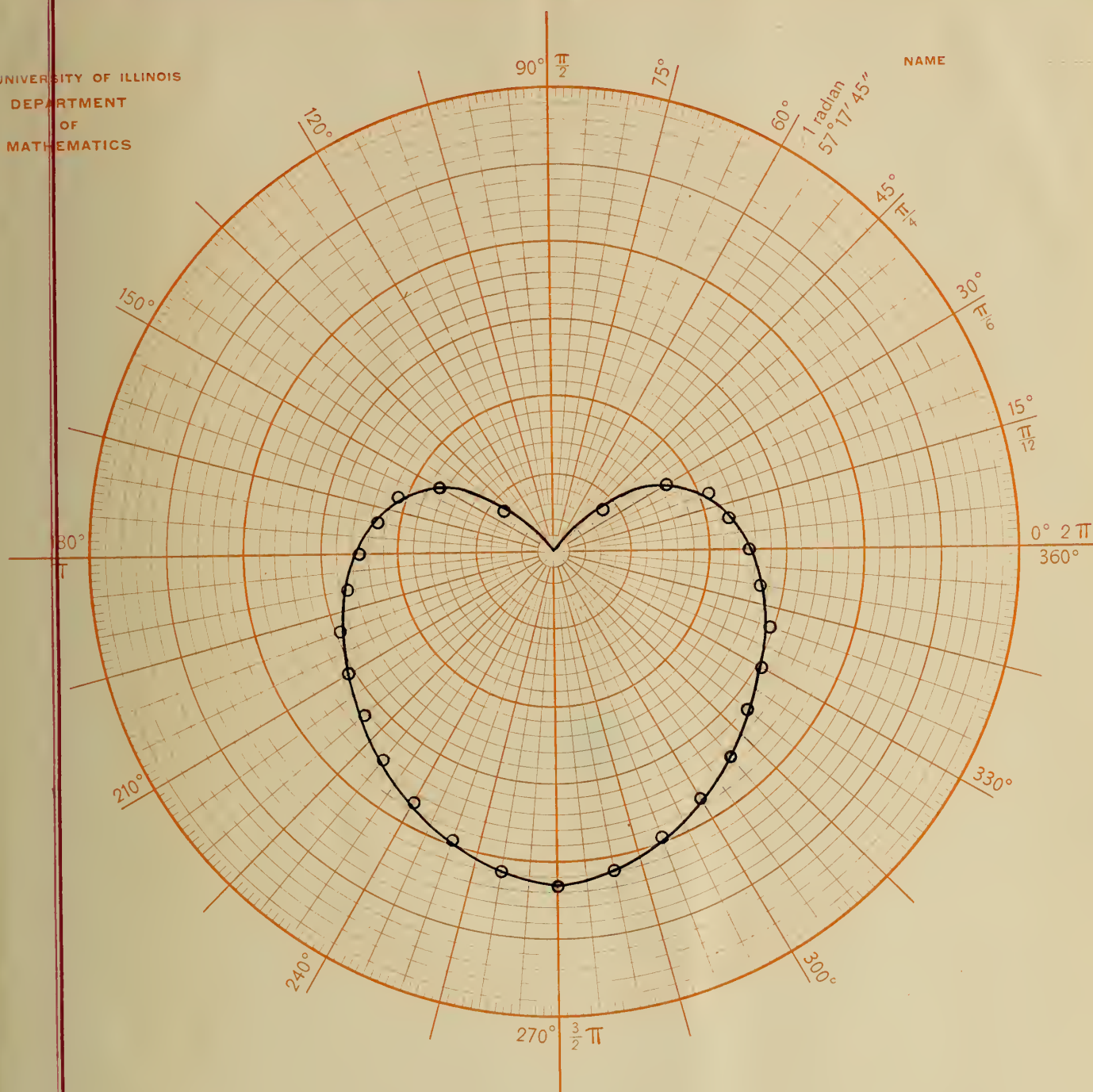
Vertical Circle

300 Degrees



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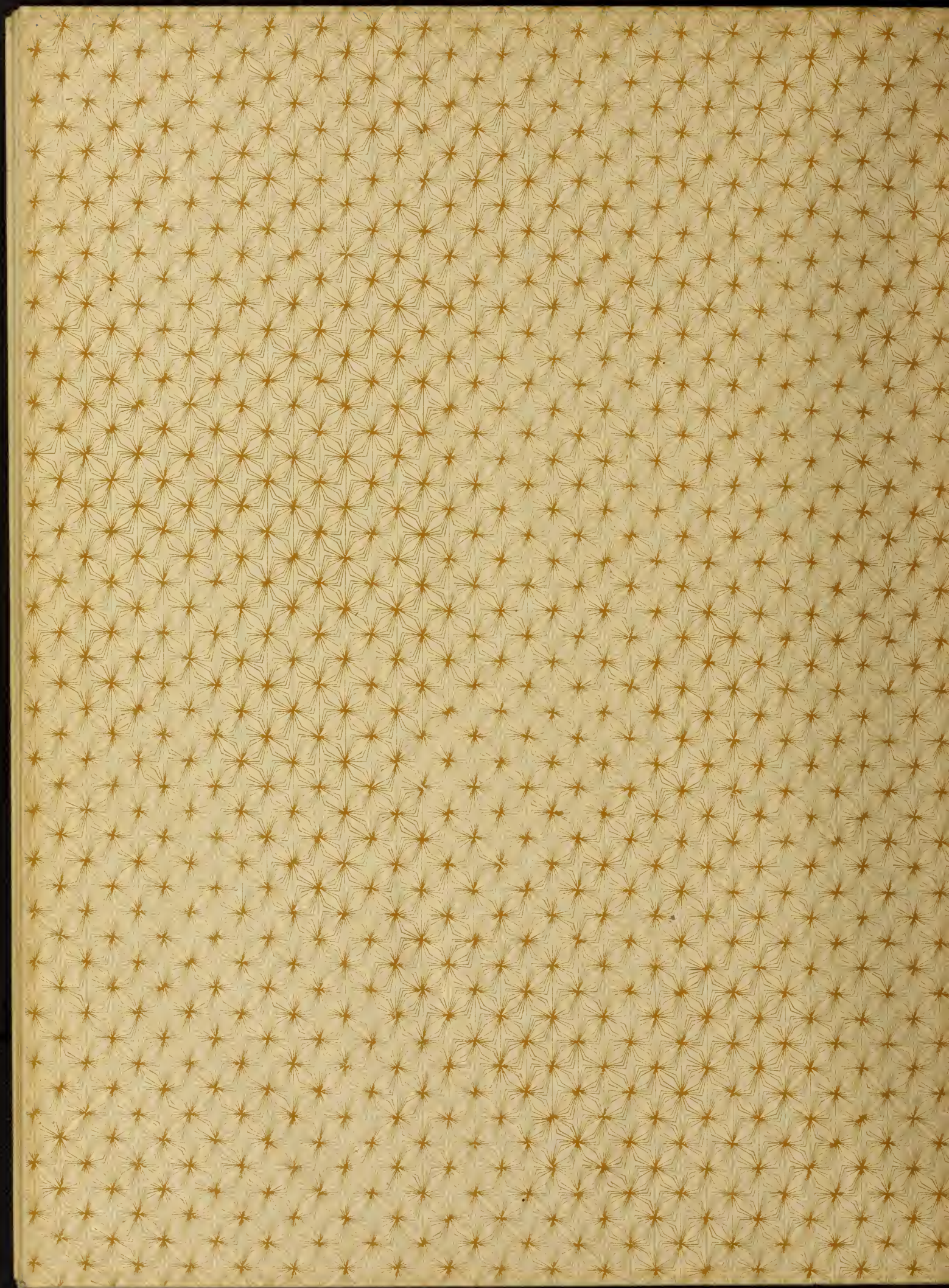
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Vertical Circle

330 Degrees







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3 0112 086857163